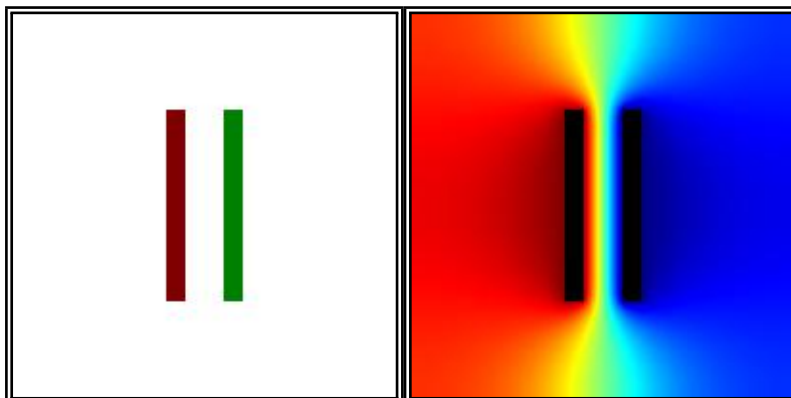


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Liebmann technical documentation

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Determination of electric field distribution in vacuum by
using relaxation method.

4

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(English version / wersja angielska)

6

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version 3

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2024.05.24

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University of Maria Curie - Skłodowska in Lublin, Poland

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98 1 Liebmann technical documentation series

- 99 1. Wyznaczanie rozkładu pola elektrostatycznego w próżni metodą relak-
100 sacyjną Liebmann. (Polish version / wersja polska)
- 101 2. Determination of electrostatic field distribution by using Liebmann relax-
102 ation method. (English version / wersja angielska)
- 103 3. Graphics. Mapping voltages to colours (colormaps)
- 104 4. Laplace equation 2D (XY). (Cartesian coordinates). Relaxation scheme
105 explained (5 - point star)
- 106 5. Laplace equation 2D (ZR). (Cylindrical coordinates). Relaxation scheme
107 explained (5 - point star)
- 108 6. Liebmann source code. (ANSI C programming language)

109 2 Versions of this document

- 110 1. version 1 - 2023.11.03
- 111 2. version 2 - 2024.01.26
- 112 3. version 3 - 2024.05.24

113 3 Explanation of symbols in calculations

- 114 • P_i - i -th mesh node
- 115 • V_i - value of electrostatic potential at node P_i . Unit - [V]
- 116 • h - mesh step (for example h_x - mesh step in x direction). Unit - [mm]
- 117 • $g_{i+/-}$ - gradient in direction i (for example $g_{1x-} = \frac{V_1 - V_{1x-}}{h_x}$. Unit - $\left[\frac{V}{mm}\right]$
- 118 • i_{row} - index of row in mesh. Values of $i_{row} = 1, 2, \dots, \text{size_row}$
- 119 • i_{col} - index of column in mesh. Values of $i_{col} = 1, 2, \dots, \text{size_col}$

120 4 Laplace equation

121 5 Solving Laplace equation using relaxation method

122 I tried to solve Laplace equation using mainly information from Pierre Grivet's
123 book (Electron Optics) - [1].

124 There are few editions of this book (1965, 1972). Second edition (1972) con-
125 tains explanation of relaxation method (page 38).

126 More generalized approaches has been drafted by James R. Nagel - [2].
127 <https://my.ece.utah.edu/~ece6340/LECTURES/Feb1/> (visited 2023-03-01).

128
129 There are also publications edited by Albert Septier: Focusing of Charged
130 Particles [3] and Applied Charged Particle Optics (part A). [4].

131 I have also found some ideas in publication of D W O Heddle: Electrostatic
132 Lens Systems [?] (especially using PC computers to solve electrostatic prob-
133 lems).

134 I have also found (brief) description of by - hand solving of Laplace equa-
135 tion by Bohdan Paszkowski - [5] (Polish edition). English translation of this book
136 also exists - [6].

137
138 I would like to thank many people, who helped me with this challenge. Espe-
139 cially prof. dr hab. Mieczysław Jałochowski (supervisor of my master's thesis),
140 who enabled me to use SIMION and MATLAB software while writing master's
141 thesis about electron optical systems at University of Maria Curie - Skłodowska
142 in Lublin in 2008. I would also thank to prof. Marcin Turek for fruitful discus-
143 sion about numerical methods. What is more, my colleague Bartosz in 2012
144 had explained me general problems with software efficiency. So he had also
145 contributed significantly to the idea of Liebmann software (especially using C
146 language).

147 **6 Mesh XY - type D**

148 $h_x = h_y = h$

149 gradient V outside a mesh does not exist

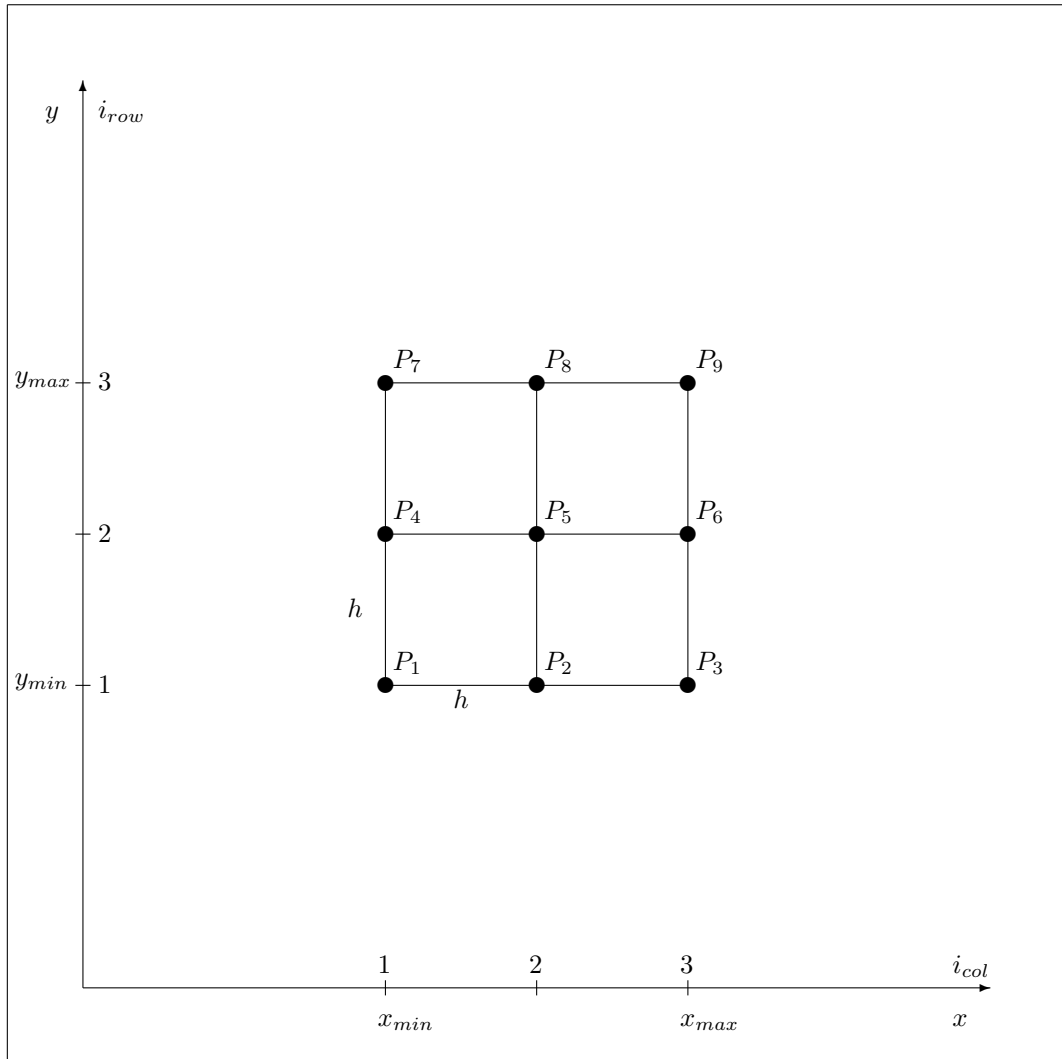


Figure 1: Mesh XY type D

150 7 Example of D-type mesh in ANSI C

151 Example of D- type mesh in ANSI C program. The mesh is analogous to B -
152 type mesh. Just $h_x = h_y = h$.

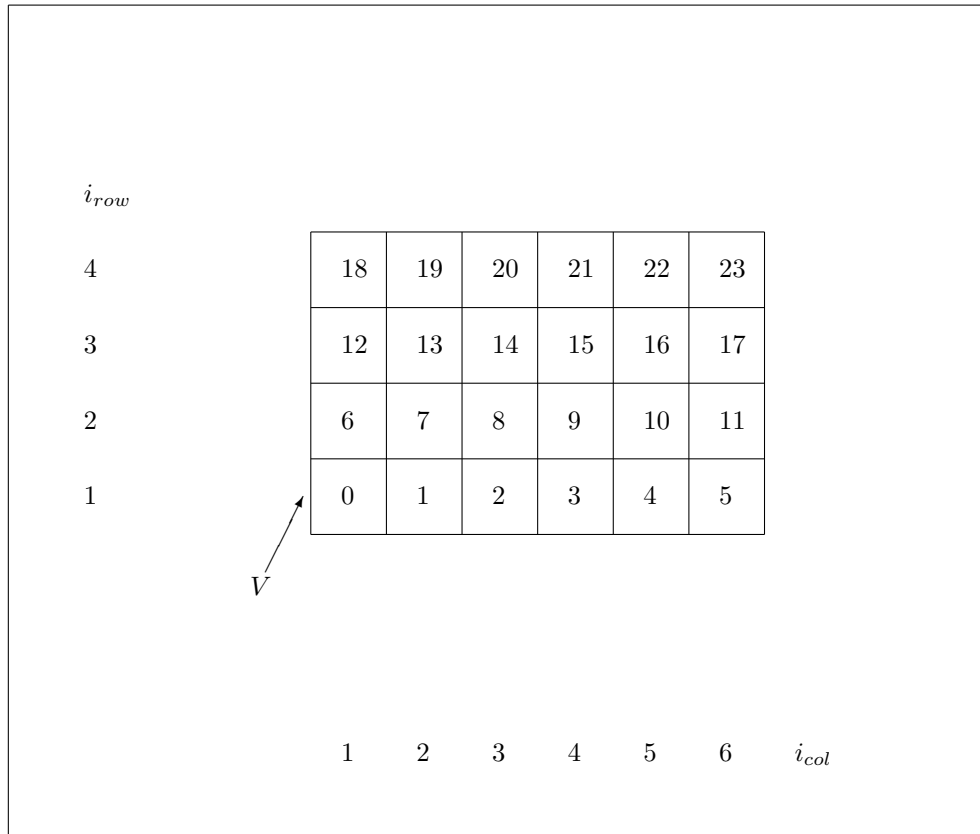


Figure 2: ANSI C - mesh XY type D

- 153 • $V \equiv \text{double* ptr_V}$
- 154 • `unsigned int size_row == 4`
- 155 • `unsigned int size_col == 6`
- 156 • `unsigned int i_row == 1, 2, ..., 4`
- 157 • `unsigned int i_col == 1, 2, ..., 6`
- 158 • `double h == 1.0 [mm]`

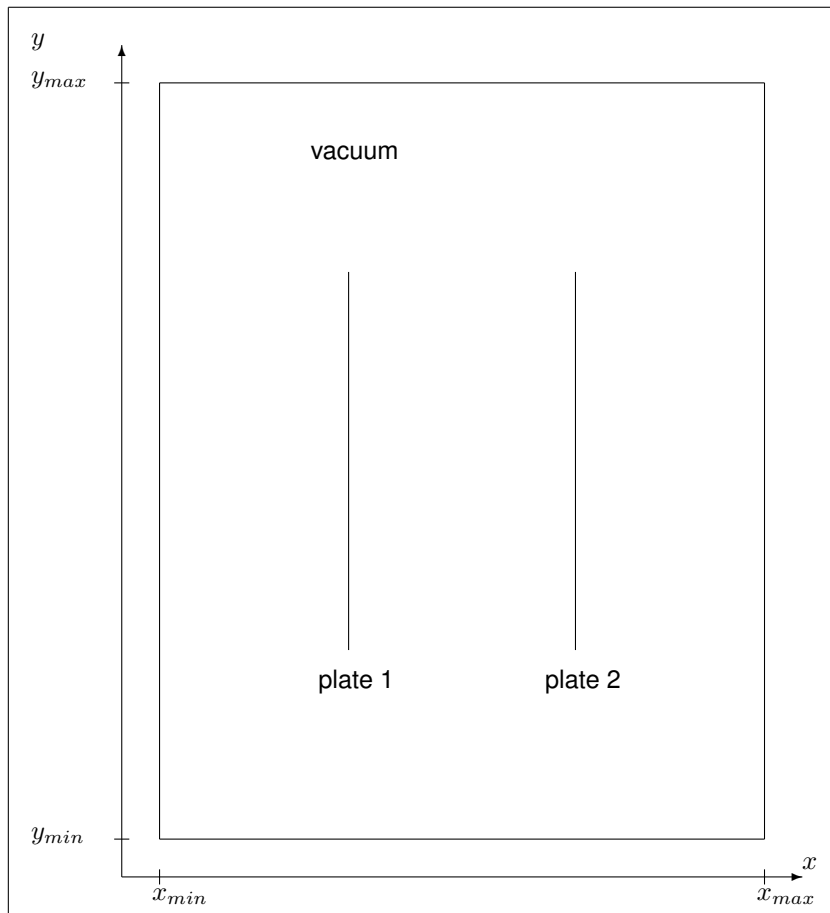


Figure 3: Example of 2D XY electrostatic problem - 2 parallel plates in vacuum

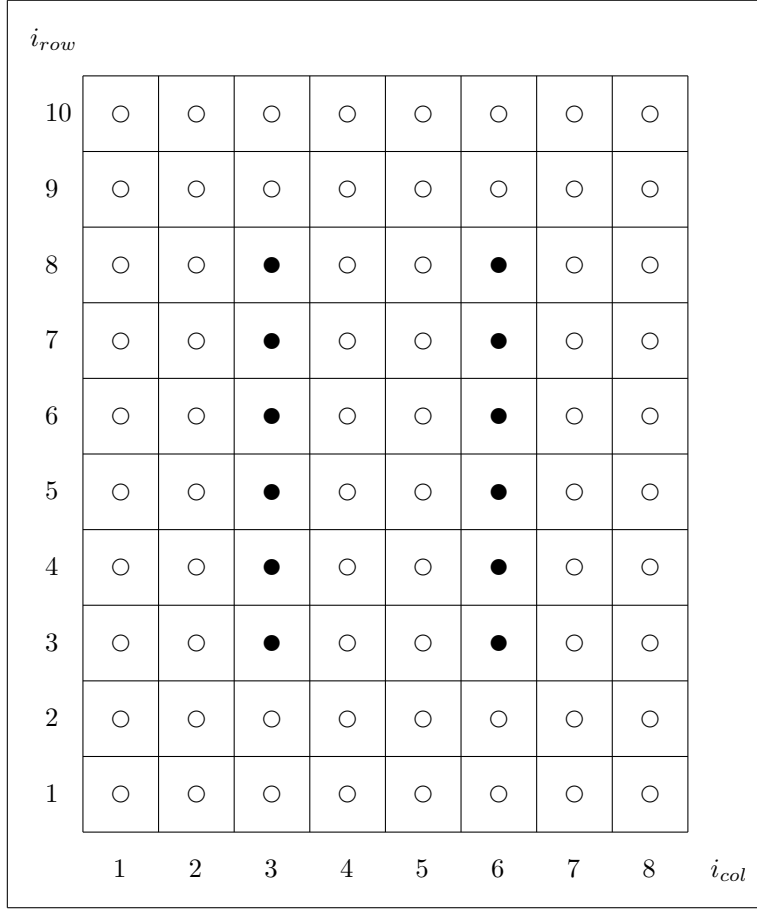


Figure 4: Nodes in sample mesh XY type D

159 $h_x = h_y = h$
 160 gradient V outside a mesh does not exist

i_{row}									
10	0	0	0	0	0	0	0	0	
9	0	0	0	0	0	0	0	0	
8	0	0	1	0	0	2	0	0	
7	0	0	1	0	0	2	0	0	
6	0	0	1	0	0	2	0	0	
5	0	0	1	0	0	2	0	0	
4	0	0	1	0	0	2	0	0	
3	0	0	1	0	0	2	0	0	
2	0	0	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	0	
	1	2	3	4	5	6	7	8	i_{col}

Figure 5: Boundaries in sample mesh XY type D

i_{row}									
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
8	0.0	0.0	+1.0	0.0	0.0	-1.0	0.0	0.0	
7	0.0	0.0	+1.0	0.0	0.0	-1.0	0.0	0.0	
6	0.0	0.0	+1.0	0.0	0.0	-1.0	0.0	0.0	
5	0.0	0.0	+1.0	0.0	0.0	-1.0	0.0	0.0	
4	0.0	0.0	+1.0	0.0	0.0	-1.0	0.0	0.0	
3	0.0	0.0	+1.0	0.0	0.0	-1.0	0.0	0.0	
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
	1	2	3	4	5	6	7	8	i_{col}

Figure 6: Values of electrostatic potential $V_{(x,y)}$ in sample mesh XY type D

i_{row}								
10	72	73	74	75	76	77	78	79
9	64	65	66	67	68	69	70	71
8	56	57	58	59	60	61	62	63
7	48	49	50	51	52	53	54	55
6	40	41	42	43	44	45	46	47
5	32	33	34	35	36	37	38	39
4	24	25	26	27	28	29	30	31
3	16	17	18	19	20	21	22	23
2	8	9	10	11	12	13	14	15
1	0	1	2	3	4	5	6	7
	1	2	3	4	5	6	7	8
								i_{col}

Figure 7: Indices in sample mesh XY type D (ANSI C)

161 Size of mesh (number of rows and columns) is:

162 `size_row == 10`

163 `size_col == 8`

164 We can have access to each node if we know its row and column: We can
165 determine its index (let's name it `icp` (index if „central” point)).

166 `icp = (i_row - 1) * size_col + i_col - 1`

167 where

168 `i_row == 1, 2, .., size_row`

169 `i_col == 1, 2, .., size_col`

170 For example, if we want to set voltage of node with index `icp`, to 1.0 Volt,
171 we can do it using pointer notation:

```
172 double* ptr_V is pointer to voltage mesh
173 *(ptr_V + icp) = +1.0;
```

174 **References**

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