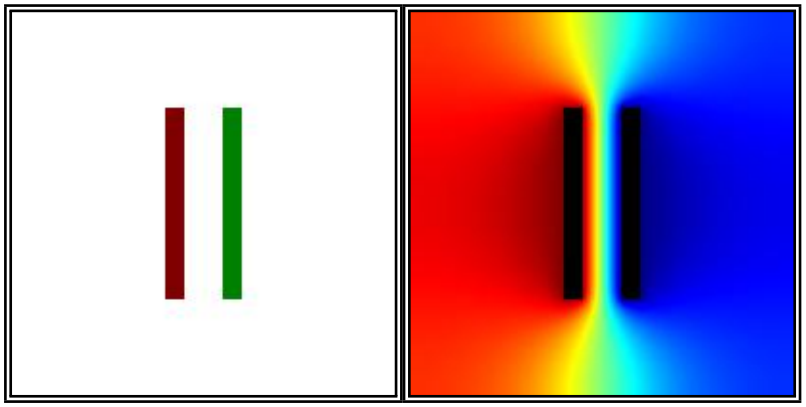
Liebmann technical documentation



Determination of electric field distribution in vacuum by using relaxation method.
(English version / wersja angielska)

author: Marcin Kulbaka
email: mkulbaka@onet.pl
project homepage: http://marcinkulbaka.prv.pl/Liebmann/index_en.html

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University of Maria Curie - Skłodowska in Lublin, Poland

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2 Versions of this document

1. version 1 - 2023.11.03
2. version 2 - 2024.01.26
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3 Explanation of symbols in calculations

- P_i - i-th mesh node
- V_i - value of electrostatic potential at node P_i. Unit - $[\mathrm{V}]$
- h - mesh step (for example h_x - mesh step in x direction). Unit - $[\mathrm{mm}]$
- $g_{i+/-}$ - gradient in direction i (for example $g_{1x-} = \frac{V_1 - V_{1x-}}{h_x}$. Unit - $\left[\frac{\mathrm{V}}{\mathrm{mm}}\right]$
- i_{row} - index of row in mesh. Values of $i_{row} = 1, 2, .., \mathrm{size\_row}$
- i_{col} - index of column in mesh. Values of $i_{col} = 1, 2, .., \mathrm{size\_col}$

4 Laplace equation

5 Solving Laplace equation using relaxation method

I tried to solve Laplace equation using mainly information from Pierre Grivet's book (Electron Optics) - [1].

There are few editions of this book (1965, 1972). Second edition (1972) contains explanation of relaxation method (page 38).

More generalized approaches has been drafted by James R. Nagel - [2]. https://my.ece.utah.edu/~ece6340/LECTURES/Feb1/ (visited 2023-03-01).

There are also publications edited by Albert Septier: Focusing of Charged Particles [3] and Applied Charged Particle Optics (part A). [4].

I have also found some ideas in publication of D W O Heddle: Electrostatic Lens Systems [**?**] (especially using PC computers to solve electrostatic problems).

I have also found (brief) description of by - hand solving of Laplace equation by Bohdan Paszkowski - [5] (Polish edition). English translation of this book also exists - [6].

I would like to thank many people, who helped me with this challenge. Especially prof. dr hab. Mieczysław Jałochowski (supervisor of my master's thesis), who enabled me to use SIMION and MATLAB software while writing master's thesis about electron optical systems at University of Maria Curie - Skłodowska in Lublin in 2008. I would also thank to prof. Marcin Turek for fruitful discussion about numerical methods. What is more, my colleague Bartosz in 2012 had explained me general problems with software efficiency. So he had also contributed significantly to the idea of Liebmann software (especially using C language).

6 Mesh XY - type D

$h_x = h_y = h$

gradient V outside a mesh does not exist

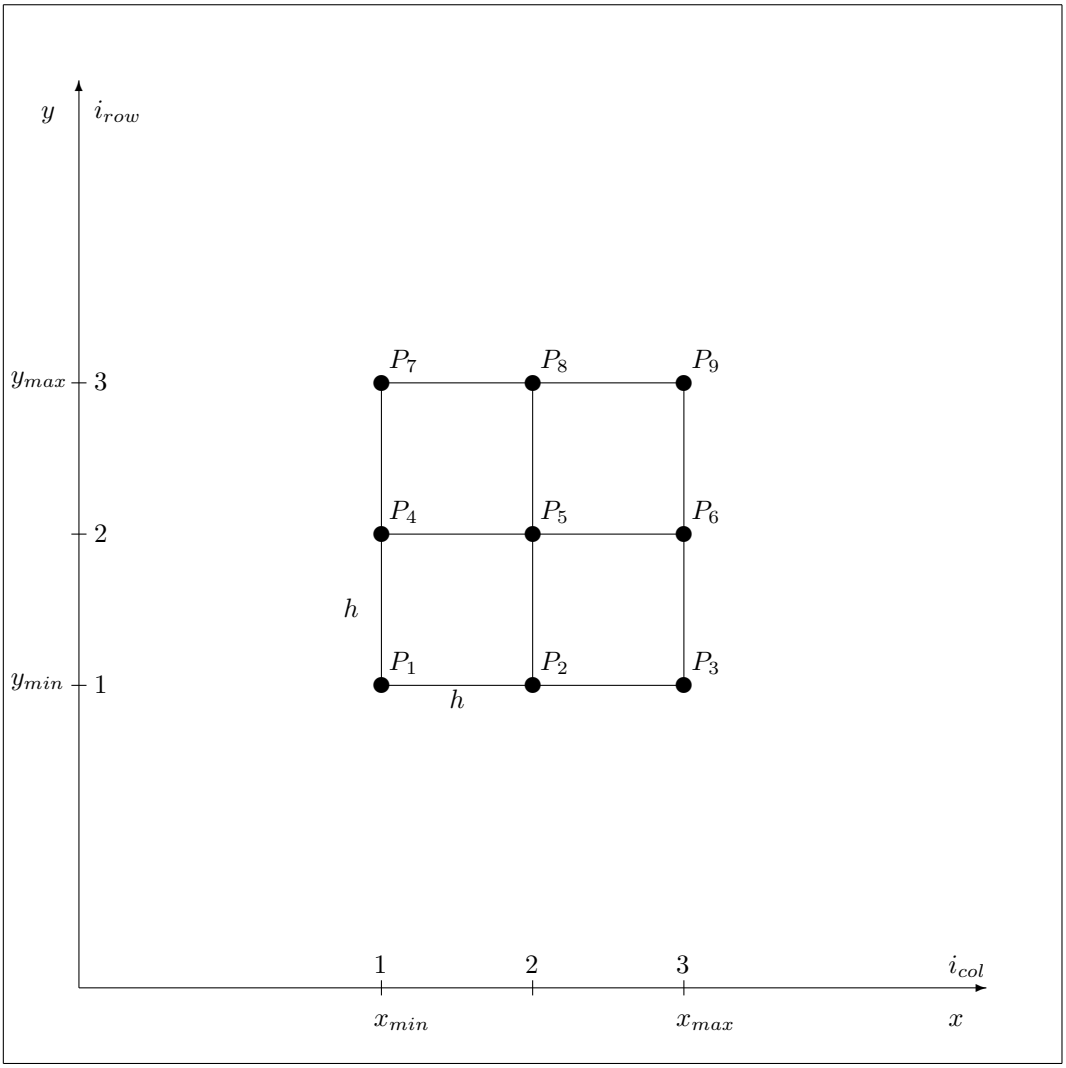

Figure 1: Mesh XY type D

7 Example of D-type mesh in ANSI C

Example of D- type mesh in ANSI C program. The mesh is analogous to B - type mesh. Just $h_x = h_y = h$.

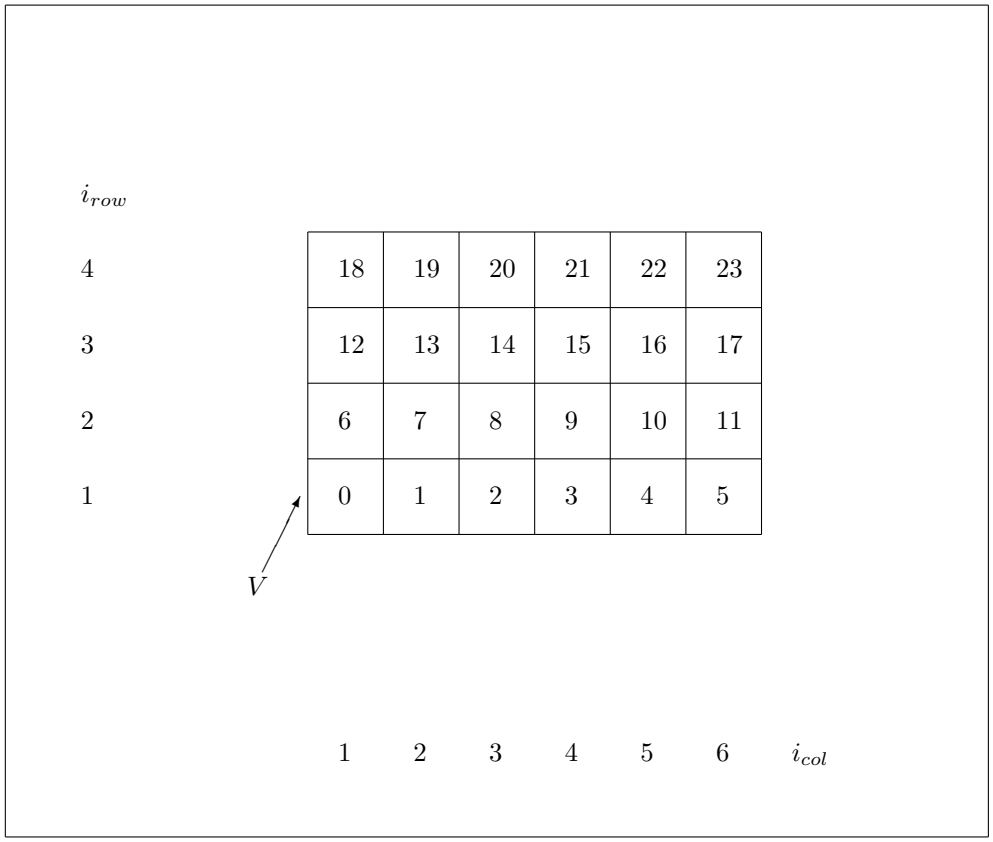

Figure 2: ANSI C - mesh XY type D

- $V \equiv$ `double* ptr_V`
- `unsigned int size_row == 4`
- `unsigned int size_col == 6`
- `unsigned int i_row == 1, 2, .., 4`
- `unsigned int i_col == 1,2, .., 6`
- `double h == 1.0 [mm]`

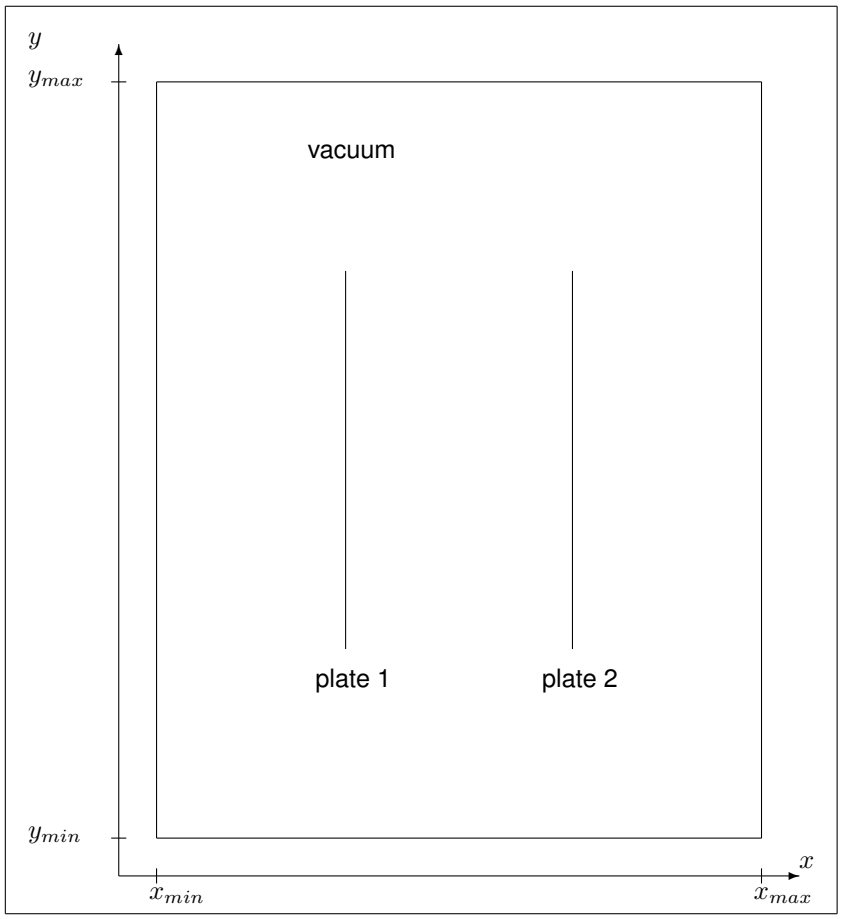

Figure 3: Example of 2D XY electrostatic problem - 2 parallel plates in vacuum

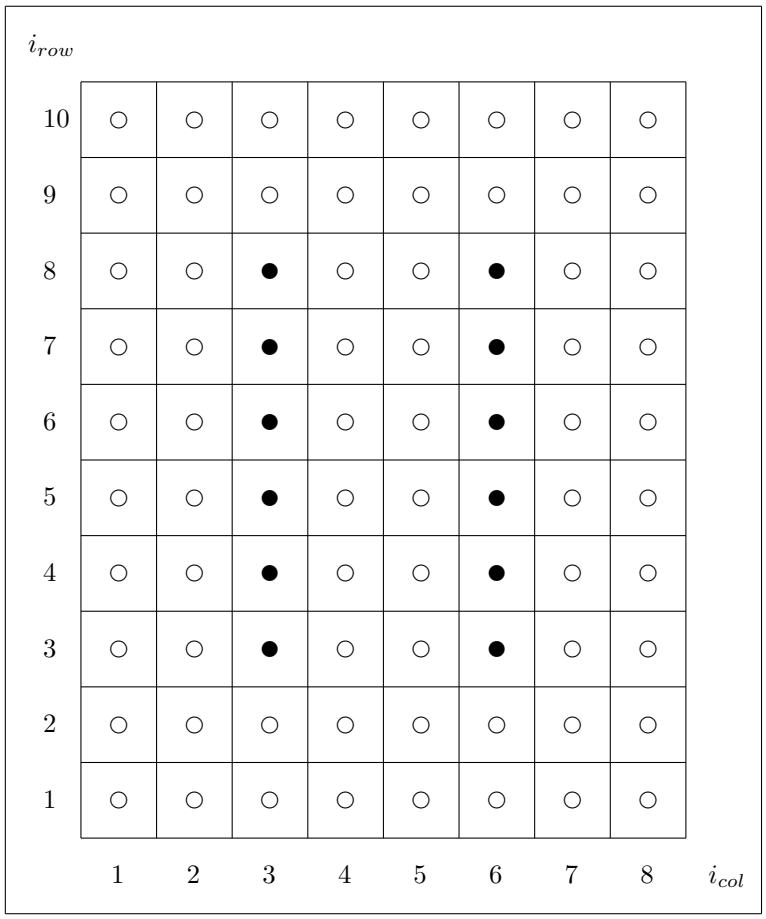

Figure 4: Nodes in sample mesh XY type D

$h_x = h_y = h$

gradient V outside a mesh does not exist

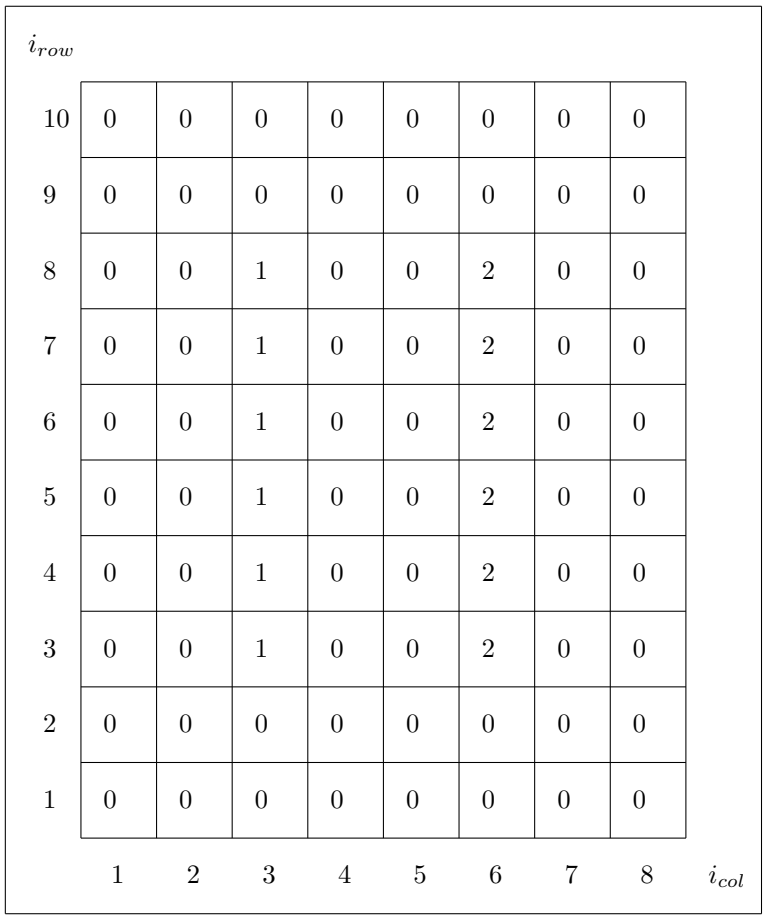

Figure 5: Boundaries in sample mesh XY type D

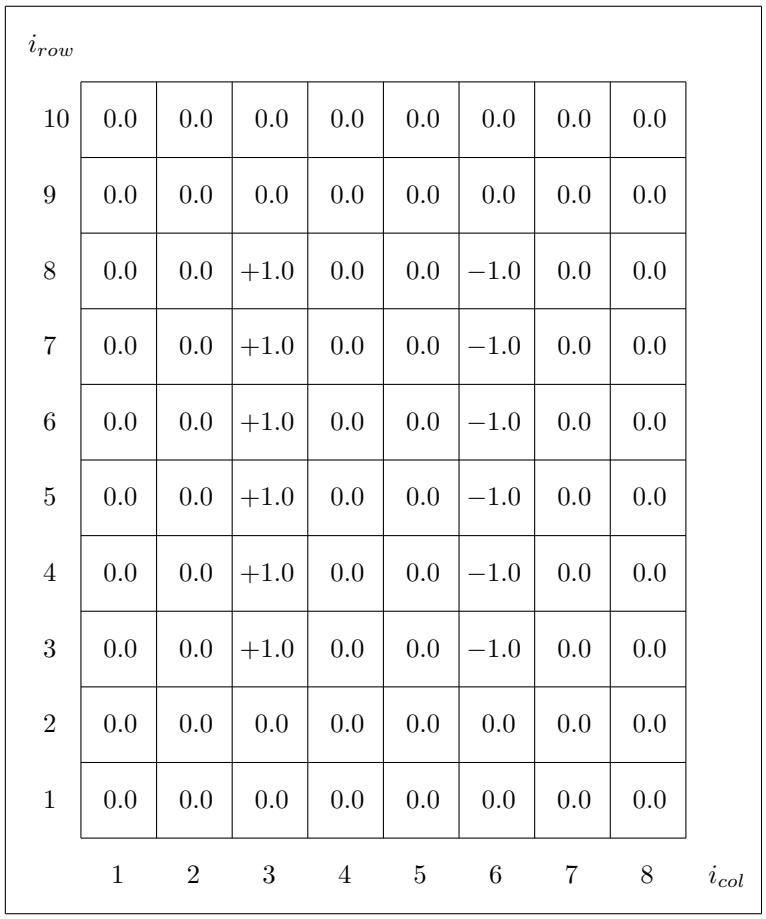

Figure 6: Values of electrostatic potential $V_{(x,y)}$ in sample mesh XY type D

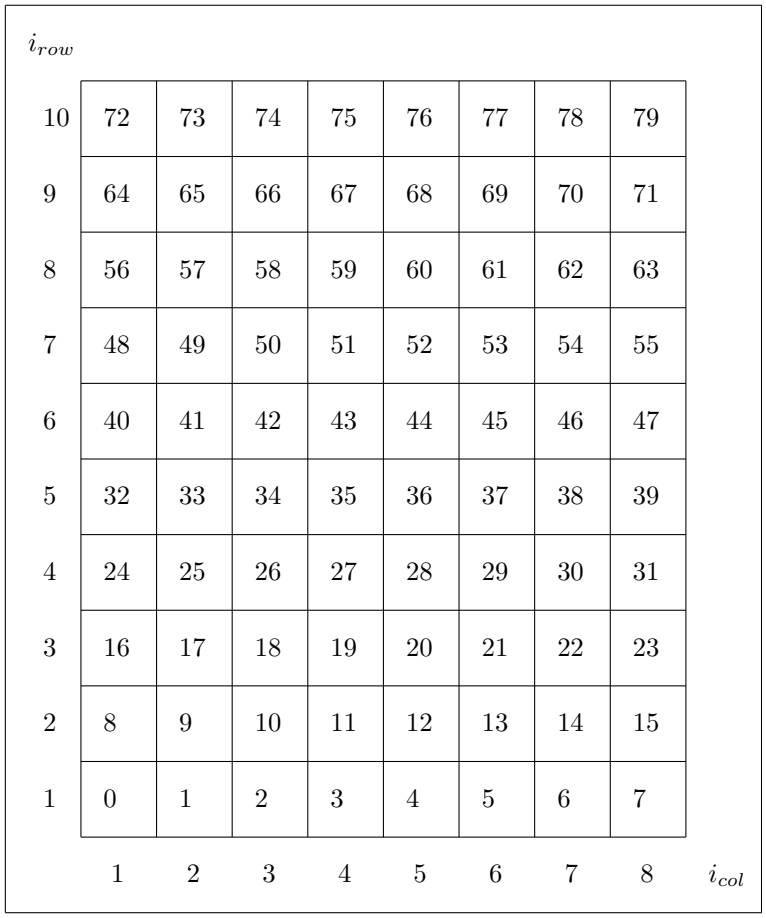

Figure 7: Indices in sample mesh XY type D (ANSI C)

Size of mesh (number of rows and columns) is:

```
size_row == 10
size_col == 8
```

We can have access to each node if we know its row and column: We can determine its index (let's name it `icp` (index if „central" point).

```
icp = (i_row - 1) * size_col + i_col - 1
```

where

```
i_row == 1, 2, .., size_row
i_col == 1, 2, .., size_col
```

For example, if we want to set voltage of node with index `icp`, to 1.0 Volt, we can do it using pointer notation:

```
172 double* ptr_V is  pointer to voltage mesh
173 *(ptr_V + icp) = +1.0;
```

References

[1] P. Grivet, *Electron Optics, Second (revised) English edition*. Pergamon Press Ltd., 1972.

[2] J. R. Nagel, "Solving the generalized poisson equation using the finite - difference method (fdm).," 2012.

[3] A. Septier(ed.), *Focusing of Charged Paticles*. New York and London, Academic Press, 1967.

[4] A. Septier(ed.), *Applied Charged Paticle Optics, part A*. New York and London, Academic Press, 1980.

[5] B. Paszkowski, *Optyka Elektronowa, wydanie II, poprawione i uzupełnione.* Państwowe Wydawnictwa Naukowo - Techniczne, Warszawa, 1965.

[6] B. Paszkowski, *Electron Optics [by] B.Paszkowski. Translated from the Polish by George Lepa. English translation edited by R. C. G. Leckey.* London, Iliffe; New York, American Elsevier Publishing Company Inc., 1968.