Abstract of the doctoral dissertation

Parallel and vectorized algorithms for solving tridiagonal Toeplitz systems of linear equations on modern multiprocessor architectures

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The primary objective of this doctoral dissertation is to develop efficient and accurate algorithms for solving linear systems of equations with a tridiagonal Toeplitz coefficient matrix, as well as more precise summation methods without loss of time efficiency. Vectorization and parallelization of these algorithms aim to utilize modern multiprocessor architectures effectively.

The first set of results involves formulating algorithms for solving the discussed systems, taking into account the architecture properties of modern computers. Implementations have been developed in the C language using interfaces that enable parallel processing, namely OpenMP (on CPU) and OpenACC (portable implementations: CPU and GPU). Novel data formats have been developed to improve memory access. The usage of different formats significantly accelerates the performance of the functions on graphics processors, including the time required for format conversion. Portable implementations between CPU and GPU, heterogeneous implementations operating on two graphics processing units, and hybrid implementations simultaneously running on CPU and GPU have been presented. Apart from the conventional column-wise data format, various data structures have been introduced. These include pure row-wise format and row-wise format with efficient conversion using cache memory and the implementation where computations are performed using cache blocks without swapping entire data arrays.

Implementations of Kahane's and Gill-Moller's algorithms have been also presented. It allows to utilize vector extensions using intrinsic functions and parallel processing using OpenMP. Accuracy results of both compensated summation algorithms are significantly better than using ordinary summation. The vectorized and parallelized functions operate significantly faster than ordinary summation. Vectorized functions run in a time similar to ordinary summation while improving the accuracy of the results. Then, this approach has been applied to the problem of numerical integration and a method for solving a specific case of tridiagonal Toeplitz linear systems. In both cases, the use of summation with corrections improves the accuracy of results.

The obtained results confirm the thesis presented in this dissertation, namely that parallelization and vectorization allow for significant acceleration of the implementation of numerical algorithms for solving systems of equations with tridiagonal Toeplitz matrices on modern processors, graphics processing units, and hybrid architectures.