Referee Report on the PhD Thesis

Markov dynamics on spaces of infinite configurations with marks

by Dominika Jasinska

The topic of the PhD Thesis under consideration is the stochastic dynamics of infinite 'particle' systems dwelling in noncompact habitats. The basic evolutionary acts studied here are the appearance and disappearance of the particles from the habitat. Such individual-based models have numerous applications in the theory of populations of various kinds. The important feature of the models introduced and studied in the Thesis is that therein each particle is characterized by the time of its presence in the population considered as a *mark*. Each such a mark increases in time with unit speed.

The Thesis consists of six chapters, of which the first two provide the biological and mathematical background, including some historical notes. The emphasis here is put on the description of configuration spaces with marks. Note that the corresponding mathematical theory is well-developed, which is reflected in the Thesis bibliography. The last chapter, an Appendix, contains technicalities.

The main results are presented in Chapters 3, 4 and 5. In Chapter 3, the Author introduces the model where the habitat is $X = \mathbb{R}^d$, $d \geq 1$, described by the following Kolmogorov operator

$$\begin{aligned} LF(\hat{\gamma}) &= \sum_{x\in\gamma} \frac{\partial}{\partial a_x} F(\hat{\gamma}) + \sum_{x\in\gamma} m(\hat{x}) \left[F(\hat{\gamma}\setminus\hat{x}) - F(\hat{\gamma})\right] \\ &+ \int_X \left[F(\hat{\gamma}\cup(x,0)) - F(\hat{\gamma})\right] \sum_{y\in\gamma} \nu_{\hat{y}}(dx), \end{aligned} \tag{1}$$

where the first term corresponds to aging - transport with unit speed along the age axis. The second and third terms correspond to the particle disappearance and appearance (here death and birth), correspondingly. These are described by the death rate m and the procreation measure ν. The form of the third term allows one to consider it as a birth term as it vanishes for an empty configuration. The main technical assumption here is that the initial state has correlation functions, which then play the central role in the description of the evolution of states. Under this assumption, by employing L as given in (1) the Author derives and solves the evolution equation for the first two correlation functions. These solutions are obtained by standard methods of the theory of transport equations. They have quite a complex structure. Unfortunately, the chapter does not contain an analysis of these solutions.

In Chapter 4, the Author introduces and studies the model where the first two terms are as in (1), whereas the third (appearance) term is taken in the form

$$\int_X \left[F(\hat{\gamma}\cup(x,0)) - F(\hat{\gamma})\right] \chi(dx),$$

which corresponds to the particle appearance independent of the existing population. The habitat is still $\mathbb{R}^d$ and the initial state is assumed to be taken from a special class of measures on $\hat{\Gamma}$. In this simpler case, the Author explicitly described (in Theorem 4.2.2) the evolution of states that preserves the mentioned class of measures. Its possible ergodicity is also proved. This result is commented in an appropriate way. I found it quite interesting and reliable, obtained by quite complex methods.

The most profound results are presented in Chapter 5, where the Author considers the same model as in Chapter 4 with the habitat X being just a locally compact Polish space. The latter does not allow here to use correlation functions as in the previous chapters. After a tedious elaboration of a special metric topology of the configuration space (Section 5.1) – followed by the equally tedious study of the corresponding Kolmogorov equation (Sections 5.2, 5.3) – the Author proves Theorem 5.4.4 which states that a martingale problem for the Kolmogorov operator L (with the domain $\mathcal{D}(L)$ constructed in Section 5.3) is well-posed. That is, it has a unique solution being a Markov process $\mathcal{X}$ corresponding to this model. The finite-dimensional marginals of its path measure are presented explicitly, and the possible ergodicity of $\mathcal{X}$ is shown to hold if the death rate m is separated away from zero. The latter is quite expectable though. The results of the Theorem 5.4.4 are quite deep. They were obtained by highly nontrivial and sophisticated methods of stochastic and functional analysis, which might prove a high level of knowledge and technical skills of the Author.

Concluding remarks

- The Thesis presents reliable and interesting results, which provide an essential contribution to the stochastic dynamic theory of infinite populations.
- The presentation is very dense but clear and the study is based on many recent publications cited in the text, proving the actuality of the treated subject.
- The obtained results were published as three research papers (one individual, two jointly with the PhD adviser) in respectable journals.

These allow me to conclude that the Thesis meets the corresponding formal and informal criteria applied to PhD Theses in mathematics, and that its Author, Dominika Jasinska, deserves the title of doctor in mathematics.

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