Agnieszka Gergont
Uniwersytet Marii Curie-Skłodowskiej

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Streszczenie rozprawy doktorskiej mgr Agnieszki Gergont pt.

On isomorphisms and isomorphic embeddings between L_1-preduals. Applications.

Supervisor: dr hab. Łukasz Piasecki

The research on isomorphisms, isomorphic embeddings and the Banach-Mazur distances belongs to the classical research trends in functional analysis. An important part of this research are attempts to estimate the distortions of isomorphic embeddings as well as obtain the exact value of the Banach-Mazur distance between two concrete Banach spaces.

The thesis is devoted to study the Banach-Mazur distances between certain spaces belonging to the class of L_1-preduals and some applications of the obtained results.

The first chapter of this thesis contains basic definitions and facts concerning selected classes of L_1-preduals.

The first part of the second chapter is an overview of the most important results concerning the Banach-Mazur distances between $C_0(K)$ spaces, while the second part is devoted to the problem of almost isometric ℓ_1-preduals.

The third chapter deals with isomorphic embeddings and isomorphisms between certain L_1-preduals. First we study the distortion $\|T\|\|T^{-1}\|$ of an isomorphic embedding T of the space c of convergent sequences into an infinite-dimensional L_1-predual X such that $(\operatorname{ext} B_{X^*})' \subset rB_{X^*}$ for some $r \in [0,1)$, that is, the set of all weak* cluster points of the set of all extreme points of the closed unit ball of the dual space X^* is contained in the closed ball of radius $r \in [0,1)$. We prove that

$$\|T\|\,\|T^{-1}\| \geqslant \frac{3-r}{1+r}.$$

Moreover, we give some examples showing that for every $r \in (0,1)$ this bound is optimal. Then, we prove a more general result in which the space c is replaced by an ℓ_1-predual hyperplane in c and we provide some examples showing that obtained estimate is optimal. In the last part of this chapter we prove that, if X is an ℓ_1-predual isomorphic to c_0, then for any isomorphism $T : X \to c_0$ (provided it exists) we have

$$\|T\|\,\|T^{-1}\| \geqslant 1 + 2r^*(X),$$

where $r^*(X) = \inf\{r > 0 : (\operatorname{ext} B_{X^*})' \subset rB_{X^*}\}$.

In chapter four we present some applications of the results obtained in chapter three. We prove some stability results for polyhedrality and extendability of compact operators in the setting of L_1-preduals. To achieve the latter goal, we additionally give a new characterization of L_1-preduals having the compact norm-preserving extension property for compact operators. Then we study topological and metric properties of the space $(\mathcal{H}, d)$, where $\mathcal{H}$ is the set of all ℓ_1-predual hyperplanes in the space c, d denotes the Banach-Mazur distance and we identify hyperplanes that are almost isometric. We prove that $(\mathcal{H}, d)$ is homeomorphic to (K, d_{ℓ_1}) with

$$K = \{x \in \ell_1 : \|x\| \leqslant 1 \text{ and } x(i) \geqslant x(i+1) \geqslant 0 \text{ for all } i \in \mathbb{N}\}$$

and $d_{\ell_1}(x, y) = \|x - y\| = \sum_{i=1}^{\infty} |x(i) - y(i)|$ for all $x, y \in K$. Then we construct a homotopy contracting $\mathcal{H}$ to c_0 along the shortest paths in $\mathcal{H}$ and we calculate their lengths. For instance, we show that the shortest path in $\mathcal{H}$ joining c and c_0 has length $\ln 4$. The last part of chapter four is devoted to the stability of the weak* fixed point property in the setting of ℓ_1-preduals. In this part of the thesis, we provide precise values of the stability constants in the class $\mathcal{H}$ and in the class $\mathcal{F}$ of all ℓ_∞^n-direct sums and c_0-direct sums of ℓ_1-predual hyperplanes in c. Then, we introduce a new stability constant based on the path metric and we provide precise values of this constant in the space $(\mathcal{H}, d)$ and $(\mathcal{F}, d)$.

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