REPORT

on the Dissertation entitled,

"Coalescence processes in continuum with applications"

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The random motion of infinite systems in the course of which the constituents can merge, attracts considerable attention. The Arratia ow introduced in [1] provides an example of this kind. In Arratia's model, an infinite number of Brownian particles move in R independently up to their collision, then merge and move together as single particles. Correspondingly, the description of this motion (and its modifications) is performed in terms of stochastic (diffusion) processes. In this work, an alternative look at this kind of motion is proposed, basing on the Kawasaki model in which point particles perform random walks (jumps) in $\mathbb{R}, d \geq 1$ with repulsion.

One of the main aims of the present work is developing and studying similar models that describe this kind of walks accompanied by coalescence. It is done by introducing an individual-based model of an infinite particle system placed in $\mathbb{R}$ in which two point particles, located at x and y, merge into a particle, located at$z \neq x; y$, with intensity $c_1(x; y; z)$. Thereafter, the new particle participates in the motion of this kind.

Base on the applications in the literature and candidates recent works, the second aim of the work is to prepare the proposed model to possible modifications as well as developing the corresponding numerical setting. As is typical for theories of this kind, the microscopic (individual-based) description provides a kind of general picture based on existential results obtained by analytic methods in suitable Banach spaces, whereas information that is more detailed can be obtained only by numerical tools. Most of them tailored to treat classical integro-differential equations of various kinds, and thus barely applicable in infinite-dimensional Banach spaces.

The dissertation consists of four parts. First, given historical developments and relations of the problem. Section 2, some basic facts and tools provided. Its first part consists mainly of discussing how metrics can introduced on configuration spaces. Relationship between the Prohorov metric and Euclidean type ones is given. Sections 3 and 4 are devoted to the construction of microscopic dynamics of the model of coalescing repulsive jumps. The contents of these two sections published by the candidate. In Section 5 a scaling from micro- to mesoscopic level of description performed. It results in a corresponding kinetic equation, which describes the evolution of density of Poisson state approximate the actual sub-Poissonian state of the system at the microscopic level. The main result of this part is given by the Theorem 5.4, which proves the continuity of the performed scaling in corresponding scale of Banach spaces. Section 6, describes the algorithm-devised for solving the kinetic equation derived in Section 5 for a specific choice of jump and coalescence intensities. It based on applying adequate boundary conditions, automatically adjusting system size and using Runge-Kutta method for numerical integration, some details given in Section 6.1.

Summary of the presented work results are:

i. Comparing Prohorov and Euclidean-based metrics on configurations spaces proving some basic metric properties (Section 2.1).

ii. Introducing the model of coalescing random jumps (Section 3.1).

iii. Proving the existence and uniqueness of local in time microscopic dynamics (Sections 3 and 4).

iv. Passing to the mesoscopic level by continuous scaling (Section 5.1).

v. Introducing extension of the model (Section 5.2).

vi. Proving the existence and uniqueness of local in time solutions to corresponding kinetic equations for both models (Section 5.2).

vii. Elaborating numerical algorithm for finding solutions to the kinetic equation for a special case of the coalescence kernel (Section 6.1).

viii. Performing numerical simulations of the system dynamics in several interesting cases and analysis of the results.

Presented work complated with 37 recent and interesting references among 5 of them are candidate's work and all of them cited in the text.

Concluding remarks: Presented work subject to The Arratia's flow in which model describes an infinite number of Brownian particles move in $\mathbb{R}$ independently up to their collision, then merge and move together as single particles. The work is developing and studying similar models that describes similar kind of motion accompanied by coalescence. The work contains number of substantial new results along its theoretical definitions and justifying numerical applications. The clarity of exposition and number of comments and meaningful explanations provides reader clear vision and getting interest to the subject. At this point I feel great pleasure to mention that getting such an interesting result because of the influence of highest calibre level PhD Supervisor and candidates hard work. Dissertation based on a series of high quality five articles of the candidate published in journals which are classified at the international level.

Without any doubt and any reservation, I conclude that this dissertation highly qualified, satisfies all the conditions to be considered for obtaining Doctor Degree in Mathematics by Krzysztof Pilorz.

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