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REPORT

on the Doctoral Thesis entitled

"Fragmentation processes in continuum with applications",
by Agnieszka Tanaś,

supervised by Prof. Dr. hab. Yuri Kozitsky,
submitted to *Maria Curie - Skłodowska* University in Lublin

Since about half-century the class of branching measure-valued processes attracted great efforts of many researchers. The state space is usually a set of positive measures on an Euclidean domain. It turns out that this class provides means of studying physical systems with infinite many elements, and therefore in an infinite-dimensional setting; see [D. A. Dawson, *Measure-Valued Markov Processes.* In: Lecture Notes in Math. **1541**, 1993], [E. B. Dynkin, *Diffusions, superdiffusions and partial differential equations*, Amer. Math. Soc., Colloquium Publications, Vol. **50**, 2002], and [Z. Li, *Measure-Valued Branching Markov Processes,* Probability and its Applications, Springer, 2011]. A specific tool in the investigation is given by the distribution functions which induces semigroups of stochastic operators in appropriate spaces.

Simultaneously, in the last decades the analysis on configuration spaces was intensively developed, requested to solve problems coming from statistical mechanic; see, e.g., [S. Albeverio, Y. Kondratiev, Y. Kozitsky, and M. Röckner, *The statistical mechanics of quantum lattice systems. A path integral approach*, EMS Tracts in Mathematics, 2009].

The subject of this thesis is placed in between the two theories mentioned above. The work presents evolution in time models for infinite populations which originated in biology. The specific properties of the models are the presence of a branching mechanism and of a random death caused by local competition. Several evolution equations are solved in spaces of functions or measures on configuration spaces and semigroups of operators are generated by the corresponding Kolmogorov operators. Two models are investigated: the fission-death system with competition and a free branching in the continuum.

We present now the **content of the thesis**.

The first chapter, **Preliminaries**, presents the general techniques and the frames used in the forthcoming results: the configuration space Γ, the topology and the Borelian structure on Γ, the temperated configurations and their properties. The sub-Poissonian measures on Γ and the Lebesgue-Poisson measure on the finite configurations are introduced in Section 1.2. Section 1.3 is devoted to the evolution equations: first, the Kolmogorov equation on functions defined on Γ and then an evolution equation for correlation functions, a substitute for the Fokker-Planck equation. In Section 1.5, the last one of the first chapter, it is presented the Thieme-Voigt perturbation theory for the generators of stochastic semigroups

The second chapter is entitled **Fission-death system with competition** and it is based on the articles [Y. Kozitsky and A. Tanaś, Evolution of an infinite fission-death system in the continuum, *J. of Math. Analysis and Appl.* **501** (2021), no. 2, 125222] and [Y. Kozitsky and A. Tanaś, Self-regulation in infinite populations with fission-death dynamics, *Phys. Lett. A* **382** (2018), 2455–2458]. In this chapter it is presented the first model, the fission-death system with competition: the states of the system are probability measures on the configuration space and each entity is subject to an independent binary fission and a state-dependent death with a rate including a competition, so, the particles interact with each other. In Section 2.1 it is first established the Kolmogorov equation describing the model and then the basic assumptions are stated. The operator L of this model (given in the formula (2.1)) has two terms, the first one corresponds to the death of a particle (under the influence of other particles), while the second term corresponds to a binary fission with a specified rate. The state space of the model is the set of all locally finite configurations of an Euclidean space. The evolution of the states of a finite system is the subject of Section 2.2. The main result is solving the associated Fokker-Planck equation on the probability measures on the finite configurations (Theorem 2.2.2). The time evolution of the states of an infinite system is the exposed in Section 2.3. The main results in solving appropriate evolution equations are obtained in Theorem 2.3.5 and Corollary 2.3.6. The last section of this chapter, Section 2.4, entitled **Mesoscopic description** contains one result (Theorem 2.4.3), showing that the time evolution from Theorem 2.3.5 preserves a "Poisson-approximable" property as it was introduced in the article [A. Barańska and Y. Kozitsky, A Widom-Rowlinson jump dynamics in the continuum, *J. Dyn. Diff. Equ.* **30** (2018), 637–665].

Chapter 3, **Free branching in the continuum**, is developed on the results from the papers [Y. Kozitsky and A. Tanaś, Evolution of states of an infinite particle system with nonlocal branching, *arXiv:2106.03483*, submitted to *J. of Evolution Equations* (2021)] and [A. Tanaś, A continuum individual based model of fragmentation: dynamics of correlation functions, *Ann. Univ. Maria Curie-Skłodowska Sect. A* **64** (2015), 73–83]. It is discussed the second model treated in the thesis, a model of branching of an infinite system of particles, located in a locally compact Polish space. The model is describes in Section 3.1, some additional assumptions on the branching kernel are introduced and discussed in this section too. Notice that there is no basic motion in this model. Subsection 3.2 is reserved to the study of the Kolmogorov operator L and the corresponding nonlinear evolution equation which is a non-local analog of the log-Laplace equation occurring in the investigation of the measure-valued superprocesses. Further on, the domain and the resolvent of L are pointed out and it turns out that L becomes a closed and densely defined operator in a convenient Banach space. Section 3.3 contains the main results of this chapter, solving the corresponding Kolmogorov equation and the Fokker-Planck equation (Theorems 3.3.1 and 3.3.3).

The proofs of the main results are presented in Chapter 4, entitled **Proofs**, the last one of the thesis. The proofs related to the fission-death model are exposed in Section 4.1. In addition, this section contains many other auxiliary results. Finally, the proofs of the results on the free branching model are exposed in Section 4.2.

The work is completed by a **Bibliography** with fifty titles cited in the text.

Concluding remarks

- This work treats a difficult subject on the Markov time evolution of infinite populations.

- The work contains a plenty of substantial new results. The proofs are delicate and use modern infinite dimensional tools. The clarity of the exposition and the numerous comments help the reader to get the real meaning of the statements. It is the style of a well established mathematician. Clearly, this is due to the influence of a PhD Supervisor of highest scientific level.

- The obtained results open the door for further investigations of interest, e.g., as it is indicated in the comment at the end of Chapter 3.

- The thesis is based on a series of four articles of the candidate, three of them written jointly with the PhD Supervisor, published or to appear in journals which are renowned at international level.

- We had the pleasure to follow a highly appreciated lecture on this topic given by Mrs. Agnieszka Tanaś in Bucharest, in November 2018, in the frame of the Potential Theory Seminar, perhaps the oldest seminar in Mathematics regularly organized in Bucharest.

It is certainly a special delight for me to give a completely positive appreciation to this work. Without any doubt I conclude that this thesis largely satisfies all the conditions to be considered as Dissertation for obtaining the Doctor Degree in Mathematics by Mrs. Agnieszka Tanaś.

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