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Abstract of doctoral dissertation

Coalescence processes in continuum with applications

There exists a broad spectrum of publications dedicated to studying dynamics of large systems of various kinds performed at different levels of mathematical sophistication. The size of the system under consideration, as well as the complexity of interactions persistent therein, predetermine the statistical/probabilistic character of the theory developed. The random motion of infinite systems in the course of which the constituents can merge, attracts considerable attention. The Arratia flow provides an example of this kind. Usually, the description of this kind of motion is performed in terms of stochastic (diffusion) processes. In the dissertation, an alternative look at this kind of motion is proposed, basing on the Kawasaki model, in which point particles perform random jumps with repulsion. One of the main aims of the work is developing and studying similar individual-based model of infinite system that describe such motion accompanied by coalescence.

The phase space of such a system is the set of all locally finite configurations. As is usual in the approach we follow, the states of the system are appropriate probability measures, in this case sub-Poissonians, which constitute a suitable class of measures associated with the well-known Poisson measure. In the dissertation, the description of this evolution is performed by solving a number of differential equations in appropriate infinite dimensional Banach spaces.

The second aim of the work is developing the corresponding numerical setting. As is typical for theories of this kind, the microscopic (individual-based) description provides a kind of general picture based on existential results obtained by analytic methods in suitable Banach spaces. The more detailed information can be obtained only by numerical tools. Most of them are tailored to treat classical integro-differential equations of various kinds, and thus are barely applicable in infinite-dimensional Banach spaces. Therefore, it might be quite natural to pass to the mesoscopic description based on kinetic equations, which rigorously can be done by an appropriate scaling procedure. The existence and uniqueness of solutions to the obtained kinetic equation is proven, but more importantly, this equation provides a numerically treatable approximation of the dynamics of the original system. An algorithm for finding numerical solutions to this equation is developed, complemented by the analysis of the results of the performed simulations. They shed some light on the details of the system behaviour, like the existence of non-trivial steady states or the emergence of propagating spatial inhomogeneities.

The results presented in the dissertation are based on the following papers:

1. Kozitsky, Y., Omelyan, I., and Pilorz, K. (2021). Jumps and coalescence in the continuum: a numerical study. Appl. Math. Comput., 390:125610.
2. Kozitsky, Y. and Pilorz, K. (2020). Random jumps and coalescence in the continuum: evolution of states of an infinite particle system. Discrete Contin. Dyn. Syst.,40(2):725–752.
3. Omelyan, I., Kozitsky, Y., and Pilorz, K. (2021). Algorithm for numerical solutions to the kinetic equation of a spatial population dynamics model with coalescence and repulsive jumps. Numer. Algorithms, 87:895–919.
4. Pilorz, K. (2016). A kinetic equation for repulsive coalescing random jumps in continuum. Ann. Univ. Mariae Curie-Skłodowska Sect. A, 70(1):47–74.

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