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Maria Curie - Skłodowska University in Lublin
Faculty of Mathematics, Physics and Computer Science

Agnieszka Tanaś

Abstract

Fragmentation processes in continuum with applications

In recent years, there has been a lot of studies of large systems of individuals interacting with each other and with the environment. Such models appear in life sciences such as biology, physics, ecology, oceanology, as well as in economics, social sciences, etc. There exists a great number of works devoted to the mathematical theory of such objects. However, the issue remained relevant nowadays is the description of such systems which takes into account the structure of the entities of the system under investigation and their time evolution.

The aim of the present thesis is to contribute to the development of the dynamical theory of infinite particle systems undergoing fragmentation. Such systems are placed in a locally compact Polish space X (trait space), where each particle is completely characterized by its location $x \in X$. Then the population dynamics consists in changing the traits of its members that includes the simplest population dynamics models which are based on two evolutionary acts: disappearance (death) of an entity and procreation (joining the population). The pure states of the system are locally finite simple configurations - subsets of X - and the dynamics of the system is described as the evolution of observables by solving the Kolmogorov equation. The general system's states are probability measures on an appropriate configuration space the evolution of which is obtained by solving the corresponding Fokker-Planck equation.

The dissertation is dedicated to introducing and studying two models of infinite particle systems. In the first model, an infinite population of point entities is placed in $X = \mathbb{R}^d$. Here, an entity with trait $x \in \mathbb{R}^d$ undergoes independent binary fission in the course of which the particle gives birth to two new particles with traits $y_1, y_2 \in \mathbb{R}^d$ and disappears afterwards. It is also subject to a state-dependent death caused by crowding – local competition. By this, the particles interact with each other. The main result here is the construction of the global in time evolution of states in a certain class of probability measures. This is done by introducing sub-Poissonian measures and then by employing the so-called correlation measures and functions. The first step to obtaining the aforementioned evolution consists of considering a finite version of the model, where I apply the Thieme-Voigt perturbation technique adapted to our purposes. Thereafter, the evolution equation for correlation functions is formulated in

the corresponding L^∞-type Banach spaces. Hence, the standard semigroup methods are not applicable here. To deal with this and to obtain a classical solution of the mentioned equation, I construct a certain (sun-dual) C_0-semigroup in appropriated Banach space, which I use to obtain a family of linear bounded operators acting from smaller to bigger spaces. I also demonstrate that the interaction as a local competition can produce a global regulating effect, i.e. the evolution of measures is obtained by identifying the solution of the evolution correlation functions equation with unique measure. I also prove that the evolution of measures preserves the sub-Poissonicity of the states and hence the self-regulation takes place. Moreover, to establish the connection of such the description with phenomenological theories, i.e. the meso- and macroscopic description, I rescaled the interaction between entities and obtain the kinetic equation corresponds to the fission-death model.

The second model presented in the thesis is devoted to an infinite system of point particles in which each particle produces at random a finite 'cloud' of new particles distributed over a locally compact Polish space X and disappears afterwards. The system of such point particles is placed in X in such a way that each compact subset of X contains only finitely many elements of the cloud, but multiple locations of particles are possible. The branching mechanism here is presented by a branching (probability) kernel, which describes the distribution of offsprings (constituting the aforementioned cloud) of the particle located at $x \in X$. It can be interpreted as a nonlocal random branching with no interactions between the particles. Like in the first model, I also employ probability measures as states of the system. However, I prove the existence and uniqueness of the solution of the corresponding Fokker-Planck equation directly, i.e., without calling correlation functions. Here, to deal with infinite configurations, I restrict the support of the considered states by imposing a condition on the branching kernel. This allows for passing to tempered configurations and solving a nonlinear evolution equation in the space of bounded continuous functions defined by the branching kernel. I define the Kolmogorov operator as a closed linear operator in an appropriate space of continuous functions and hence obtain unique classical solvability of the Kolmogorov equation by constructing a C_0-semigroup generated by this operator.

The results contained in the thesis are based on the following papers.

[1] Y. Kozitsky and A. Tanaś, *Evolution of states of an infinite fission-death system*, arXiv:1804.01556, to appear in Journal of Mathematical Analysis and Applications (accepted) (2018).

[2] Y. Kozitsky and A. Tanaś, *Self-regulation in infinite populations with fission-death dynamics*, Phys. Lett. A **382** (2018), no. 35, 2455–2458.

[3] Y. Kozitsky and A. Tanaś, *Evolution of states of an infinite particle system with nonlocal branching*, submitted to Journal of Evolution Equations (2021).

[4] A. Tanaś, *A continuum individual based model of fragmentation: dynamics of correlation functions*, Ann. Univ. Mariae Curie-Skłodowska Sect. A **64** (2015), no. 2, 73-83.

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