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PhD thesis summary

Fixed point theorems for semigroups of nonlinear mappings in weak topologies

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This PhD thesis presents results related to the hypothesis stated around 1970 by A. T.-M. Lau, saying that specific nonexpansive representations of the so-called left amenable semigroups have a common fixed point [1]. Although our research did not provide the definite answer to the validity of the hypothesis, it turned out that there are some special cases for which the hypothesis is true, among others, for commuting semigroups and firmly nonexpansive representations. Especially, the new property has been shown, which – if possesed by the semigroup – guarantees that Lau's hypothesis is fulfilled.

Beside the results related to the Lau hypothesis, the formed techniques allowed to acquire new theorems in other domains, i.a. for affine mappings and nonexpansive retractions. In this context two results are worth noting: proving Xu and Yamada's conjecture [2] about the rate of convergence of the iteration of the average affine mapping, and generalizing Bruck's theorem about the nonexpansive retraction of commuting semigroups [3].

The thesis begins with showing the property shared by left amenable and left reversible semigroups (which is needed in the sequel), and also introduces the generalized versions of asymptotical regularity that go beyond metric spaces. Also the first fixed point results arise there.

The next part focuses on theorems about retractions on fixed point sets of mappings. The results using Bruck's theorems can be found there and others, based on different techniques, e.g. the theorem which utilizes the Tchebyshev centre, inspired by [4].

The thesis ends with the summary of known special results of the Lau hypothesis, the old and new ones.

Some of the results were published in [5], [6], [7].

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2) H.-K. Xu, I. Yamada, Asymptotic regularity of linear power bounded operators, Taiwanese Journal Of Mathematics Vol. 10, No. 2, 417-429, February 2006.
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4) U. Bader, T. Gelander, N. Monod, A fixed point theorem for L_1 spaces, Inventiones Mathematicae 189 (2012), 143-148.
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7) S. Borzdyński, Common fixed point theorems for nonexpansive mappings using the lower semicontinuity property, Colloquium Mathematicum 154 (2018), 157-165.

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