Rzeszów, 30 maja 2019 r.

Abstract of the doctoral thesis

Existence problems for infinite systems of differential equations in Banach sequence spaces

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The doctoral thesis is devoted to existence theorems for infinite systems of ordinary differential equations. Solutions of those systems are looked for in classical Banach sequence spaces as well as in more extensive spaces i.e. in spaces of tempered sequences.

The main tool utilized in the thesis is the technique of measures of noncompactness applied to Banach sequence spaces. We use here some suitable formulas expressing those measures in classical sequence spaces and, on the basis of those measures we construct new measures of noncompactness in the spaces of tempered sequences. Additionally, due to the fact, that infinite systems of ordinary differential equations can be treated as a particular case of differential equations in Banach (sequence) spaces, while considering those systems we use of the results obtained in the general theory of differential equations in those spaces. On the other hand infinite systems of differential equations require the application of special research methods related to the specificity of those systems.

The presented Ph.D. thesis creates a continuation of the above mentioned study of infinite systems of differential equations. Particularly, we consider here systems formatted as nonlinear perturbations of linear systems. Those linear systems have the form of so-called lower diagonal systems or upper diagonal systems. Taking into account that those systems are infinite, we consider the mentioned "linear parts" as the so-called parts with the constant width which means that the linear part appearing in each equation consists of finite number of terms, where that number does not exceed of a priori given number. That constraint is a result of a large number of technical difficulties appearing in our considerations.

In the thesis we present a lot of results concerning the existence of solutions of infinite systems of differential equations. Those results are formulated under assumptions which can be easily verified, if we consider concrete infinite systems of differential equations. In the thesis we consider a lot of particular situations related to those types of infinite systems of differential equations which are applied to various practical problems, described in the classical monograph of K. Deimling [4] concerning differential equations in Banach spaces.

Theorems on existence of solutions of infinite systems of differential equations, which are given in the Ph. D thesis, are illustrated by the appropriate examples.

The results contained in the thesis are based mainly on papers [1, 2, 3], which are entirely devoted to the theory of infinite systems of differential equations with the use of tools of the theory of measures of noncompactness.

References

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