PhD Thesis Report

Rinaldo M. Colombo
INdAM Unit
Department of Information Engineering
University of Brescia

August 26th, 2019

Title:	Conservation laws in the modelling of collective phenomena
Candidate:	Nikodem Dymski
Supervisors:	Paola Goatin & Massimiliano D. Rosini

Report

This thesis consists of 6 chapters and 2 appendixes. The results therein are also in the articles [2, 3, 4], in the preprint [6] and in the proceedings [5]. The articles are published in scientific journals of renowned international level and, to get there, already passed a careful refereeing process.

Chapter 1

Here we find an introduction to the modeling of vehicular traffic by means of conservation laws. Relevant literature is cited and some hints to other modeling frameworks are provided. There is an effort to introduce and justify the fundamental key relations, such as the one connecting density, flow and speed.

Chapter 2

The reader is introduced to the models that most attracted the interest of the candidate, essentially the LWR [8, 9] and ARZ [1, 10] models, the latter also with phase transitions [7]. General definitions (e.g., of weak entropy solution) are provided in the particular cases under consideration.

Chapter 3

This very short chapter is of introductory nature and concerns the constrained LWR model. Its role is to prepare the reader to the extension of unilateral constraints achieved in the later chapters with reference to more complex models.

Chapter 4

Describes the results published in [4], namely the proof of the existence of solutions to the ARZ model under point constraints. The technique exploited essentially relies on wave front tracking, as in the rest of the thesis.

Chapter 5

This is the longest chapter. It is devoted to the study of phase transitions models, see [2, 3]. The so called *"Introduction"* actually considers the case PT^a, while § 5.1 deals with PT^p. The presence of different phases, justified by different qualitative behaviors of vehicular traffic, hinders the use of standard analytic techniques in dealing with basic issues such as the existence and continuous dependence of solutions.

Chapter 6

The results presented here are those published in [5, 6]. They deal with the effects of point constraints on the flow applied to a point moving along a network. Both the LWR and the phase transition models are considered.

Appendices

They collect various technical details. Their ordering is not of great help to the reader.

Evaluation

The results presented in this thesis deserved publication on scientific journals of international level. I personally agree with the positive evaluations by the referees: I believe that these results are of interest for the community of researchers working on the macroscopic modeling of vehicular traffic. More specifically, I believe that this thesis does indeed solve problems in the theory of partial differential equations that are of interest in applications. Therefore, I recommend that the candidate be awarded the title of *Doctor Philosophiæ*.

I hereby express my **favorable opinion** towards the thesis defense.

Hints for Further Discussions

This work naturally rises a variety of questions, most of which could also be considered as hints for further works. I list some of them here.

1. A recurrent key difficulty in the analytic results obtained is hidden in *exchanging the limits*. Constraints are inequalities to be satisfied by traces of the (exact) solution. Exact solutions are constructed as limits of approximate solutions. Hence, almost every existence proof ends showing that the trace of the limit is the limit of the traces. Does this thesis point to a somewhat general approach that makes this *exchanging the limits* possible?

2. The most traditional applications of conservation laws are strictly related to fluid dynamics, where entropy plays a key role, also from the analytic point of view. What role does entropy have in the applications of conservation laws to vehicular traffic?

3. From the point of view of applications, the continuous dependence of solutions from the data and their stability with respect to perturbations of the equations are relevant. To which extent can we expect that results of these kinds hold in the modeling of vehicular traffic under constraints?

4. As this thesis shows, the presence of a constraint, in general, increases the number of waves. Consider traffic flowing along a network containing a loop: can the presence of a constraint produce so many waves that the total variation of the solution blows up?

5. The title of the thesis suggests that conservation laws can be applied also to modeling frameworks not necessarily related to vehicular traffic, Can also the techniques developed in this thesis be brought to theseq other modeling situations?

6. A recurrent functional setting in this thesis is provided by the space **BV** of functions of bounded variation. Does the restriction to this space have consequences on the applicability of the various results to real situations? Do the total variations in space, in time and in both play similar roles?

References

[1] A. Aw and M. Rascle. Resurrection of "second order" models of traffic flow. *SIAM J. Appl. Math.*, 60(3):916–938 (electronic), 2000.

[2] M. Benyahia, C. Donadello, N. Dymski, and M. D. Rosini. An existence result for a constrained two-phase transition model with metastable phase for vehicular traffic. *NoDEA Nonlinear Differential Equations Appl.*, 25(5):Art. 48, 42, 2018.

[3] E. Dal Santo, M. D. Rosini, N. Dymski, and M. Benyahia. General phase transition models for vehicular traffic with point constraints on the flow. *Math. Methods Appl. Sci.*, 40(18):6623–6641, 2017.

[4] N. S. Dymski, P. Goatin, and M. D. Rosini. Existence of **BV** solutions for a non-conservative constrained Aw-Rascle-Zhang model for vehicular traffic. *J. Math. Anal. Appl.*, 467(1):45–66, 2018.

[5] N. S. Dymski, P. Goatin, and M. D. Rosini. The Riemann problem for a general phase transition model on networks. In C. Klingenberg and M. Westdickenberg, editors, *Theory, Numerics and Applications of Hyperbolic Problems I. HYP 2016*, volume 236 of *Springer Proceedings in Mathematics & Statistics*. Springer, Cham, 2018.

[6] N. S. Dymski, P. Goatin, and M. D. Rosini. Modeling moving bottlenecks on road networks. Preprint, Jan. 2019.

[7] P. Goatin. The Aw-Rascle vehicular traffic flow model with phase transitions. *Math. Comput. Modelling*, 44(3-4):287–303, 2006.

[8] M. J. Lighthill and G. B. Whitham. On kinematic waves. II. A theory of traffic flow on long crowded roads. *Proc. Roy. Soc. London. Ser. A.*, 229:317–345, 1955.

[9] P. I. Richards. Shock waves on the highway. *Operations Res.*, 4:42–51, 1956.

[10] H. M. Zhang. A non-equilibrium traffic model devoid of gas-like behavior. *Transportation Research Part B*, 236:275–290, 2002.

Brescia, August 26th, 2019

Rinaldo M. Colombo