Report on the PhD Thesis

CONSERVATION LAWS IN THE MODELLING OF COLLECTIVE PHENOMENA

by Nikodem Dymski

to be discussed at the

University Marie Curie-Skłodowska of Lublin

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The Ph.D. thesis by N. Dymski deals with the mathematical analysis of some models of vehicular flows; some applications to crowds dynamics are also mentioned, whence the title that gathers both topics. The thesis is essentially based on five papers [10, 34, 35, 42, 43] that the author wrote in collaboration with other mathematicians, in particular with his advisors; one [34] of the papers is published in the proceedings of a conference, another one [43] is still in preprint form. The numbering of the references here and in the following is the one of the bibliography of the thesis.

The topic of the thesis is very focused: the author considers hyperbolic systems of partial differential equations, possibly including one ordinary differential equation, in most cases with a *constraint*: indeed, the last requirement strongly characterizes the whole research. The aim is to establish the existence and properties of solutions. In some cases the author only deals with the Riemann problem; in two other cases he studies the (much more difficult) Cauchy problem. The latter is well-known to be a very hard and technical problem; Tables A.1 on page 129 and just a look at the proof of Proposition 5.7 in Section A.5 give an approximate idea of what I wrote.

The manuscript consists of six chapters, two appendices and an exhaustive bibliography.

Chapter 1 introduces the main issues in traffic flows modeling, with mention to micro-, meso- and macroscopic models; a short account to the content of the thesis is also provided.

Chapter 2 shows in detail the macroscopic models to be considered in the following and their Riemann solvers. These models are, namely, the LWR (from Lighthill-Whitham-Richards), the ARZ (Aw-Rascle-Zhang) and the PT (phase transition), whose original formulation was given by Colombo in [28] and then extended in [15, 53]. The first two models are rather well-known and then the author mainly focuses on the latter. The PT model is presented in broad generality, both in the case of metastable and non-metastable phases; this means that the free and the congested zones do intersect or not, respectively.

Chapter 3 briefly accounts for the case of a single constrained equation, and it summarizes results obtained in [29] by Colombo and Goatin, then extended in some other papers.

The core of the thesis is in Chapters 4 and 5, where the author fully shows his mastering of the wave-front tracking technique, in particular as far as interaction estimates and the control of the growth of the total variation is concerned. Both chapters are highly original, though exploiting several techniques already existing in the literature, and very demanding for the reader. To same extent, they are rather similar both for results and for the techniques.

Chapter 4 is largely taken from the paper [42] by the author and collaborators. The starting point is the paper [4], where the ARZ model with flow constraint was considered in the framework of a conservative Riemann solver. The nonconservative case was then tackled in [50] under restrictive assumptions. The aim of [42] and of this chapter is to consider the latter case in full generality.

Chapter 5 contains results published in [10, 35] and is concerned with the Cauchy problem for PT^p model with flow constraint and metastable phase. A discussion of the Riemann solver for the PT^a case is also provided together with the consistence of the corresponding Riemann solver.

Chapter 6 illustrates some recent results obtained by the author in [34, 43] about moving bottlenecks (for instance, a single bus) along a road and the PT model on star networks. The results are interesting and original, though not so deep as those presented in Chapters 4 and 5.

From a mathematical point of view, all results are correct and original, to the best of my knowledge. In spite of the high level of technicalities of some proofs, the author did his best to organize the presentation in the best way; several figures are disseminated in the text and are always very well done and helpful.

The style of the thesis is good, but could be a bit better. The English text, even if sufficiently clear and not hindering the comprehension, is sometimes awkward. There are several typographical incongruences and some words are misspelled. Also several names are misspelled: Lipshitz for Lipschitz (pages 15, 44), Heavyside for Heaviside (page 17), Kruzkov for Kruzhkov (page 44). Gauss-Green formula is also sometimes mentioned as Green-Gauss formula (e.g., pages 57 and 58).

The bibliography is written using several different styles and so it is very far from being homogeneous. Among several flaws, items [54] and [55] seem the same item; I found it curious not to cite the original fundamental paper of P.D. Lax (please insert also the first letter of the second name), a real piece of history of mathematics, but simply make reference to his Selected Papers.

Here follows a list of comments.

(i) On page 4 the author writes that *condition* **(A.1)** *guarantees the conservation of cars.* It seems to me that this is due to the conservation form of the equations; hyperbolicity is not sufficient, in general.

(ii) Probably in (1.4) v_t should take the place of the first v.

(iii) On page 10 the author mention the *pressure* in Payne-Whitham model, but I could not see where it was introduced.

(iv) On page 11 the author writes that *the density ρ is the unique independent variable, so it is 1D.* Usually 1D refers to the space dimension rather than to the number of equations.

(v) On page 14 the author quotes several papers referring to the state-of-art. Indeed, these papers range from year 2011 fo 2014; they are surely very interesting, but maybe not so up-to-date.

(vi) I appreciated the detailed list of drawbacks of the LWR model provided on page 21. I regret that the same was not done for the other models under study; this would have been very useful to the reader. I point out that I do not follow the author when he writes that the LWR model recognizes the desired velocity of each vehicle.

(vii) Probably the requirement that $p(\rho) > 0$ in (2.12) is implied by the other assumptions. Indeed, it is missing in (5.8). The line above: should upstream replace downstream?

(viii) On page 29, I found the motivations for models PT^a and PT^p a bit vague. In particular the author writes that both cases are motivated by practical reasons, but does not specify which. Since the model PT^a is intensively studied in the rest of the thesis, some motivations to the choice of v^a, see page 30, had better to be provided, possibly together with a general comparison of the two models.

(ix) Figure 2.2 is very useful; however, it is not immediately clear what the thick lines represent; indeed, this is explained in the following pages.

(x) On page 34 the author writes that for *modeling consistency, ... the flux ... has at most one inflection point.* I do not follow the author about the consistency of the modeling.

(xi) On page 40 the author states the important notion of consistence of a Riemann solver claiming that, in general, it is a necessary condition for the L^1-continuity of the semigroup generated by the wave-front tracking algorithm. However on page 49 I could find no information about the consistence of the Riemann solver $\mathcal{CRS}_{\mathrm{ARZ}}$. Indeed, the L^1-continuity is missing in Theorem 4.1. On the contrary, this information

is provided for the two Riemann solvers on page 67 in the useful Table 5.1; they are not consistent by Proposition 5.1. Nevertheless this does not prevent the study of the Cauchy problem. About the PT^p case with constraint and metastable phase, see Section 5.2, this information could have been useful.

(xii) The title of Section 5.1 is a bit misleading, since it almost only deals with the PT^a case; indeed, Section 5.2 deals with the PT^p case.

(xiii) The definition of $\delta(t)$, though understandable, seems missing in Proposition 5.6.

(xiv) In page 86 the author mentions the Temple functional. However, no motivation for this name is provided, nor Temple is cited in the bibliography.

(xv) On (6.3) it seems that the definition of α is missing.

(xvi) The title of Appendix A seems a bit misleading, since it also deals with proofs contained in Section 5. Indeed, only Sections A.1 and A.2 deal with Section 2.4.3. Sections A.3 to A.6 (in particular the very important Section A.5) would deserve a different appendix to be put in the due value. On the contrary, Appendix B comes back to material in Section 4.

Here follow some questions and other issues.

1. On page 3, the authors claims, referring to microscopic models, that *online simulations are computationally very demanding.* However, a reference is missing. Can the author provide or quote estimates of computing time for micro and macroscopic models, corresponding to the same phenomenon?

2. In the first two chapters the authors mentions multilane and multipopulation models. How could these models be considered in the presence of a constrained flow?

3. To what extent do the models considered by the author allow for phantom jams? That was one of the drawback of the LWR model on page 21.

4. About model PT^a, one could wonder whether the dependence of the velocity v^a on y should be linear or if one could substitute y for a general function $g(y)$ vanishing at 0. Are there experimental motivations to the introduction of such a velocity v^a?

5. The constraints in this thesis give rise to nonclassical waves. Even if this is not the subject of this thesis, I wonder whether the existence of traveling waves profiles for these waves in this special context has been considered in the literature.

6. On page 58, it is not completely clear to me why $\partial_t \rho^n + \partial_x f(U^n) = 0$ in weak sense. The same occurs (not explicitly) also in the corresponding proof on page 98.

7. By comparing the two case studies in Section 4.4 (for the constrained ARZ model) and 5.2.3 (for the constrained PT^p model with metastable phase), one notices that the plots on the right in Figures 4.3 and 5.9 are very similar. Indeed, in the case of a toll gate, and then with high car densities, one wonders whether a different modeling of the free flow is really important. Notice that the exit times t_L on pages 61 and 82 are equal. Has it been done a numerical comparison of the two models, at least in this specific case?

8. On page 64 it is not clear the range of application of Proposition 5.1, which is stated in a completely general way. The proof, indeed, exploits the framework previously introduced, see Figure 2.2.

9. A general question concerns the use of the different models, apart from their analytical treatment: for instance, considering the ARZ, PT^a and PT^p models (with and without metastable phase), which one does better represent the experimental data? Or, when is it better to use one of them instead of the others?

As a conclusion, my opinion is that the Ph.D. thesis of N. Dymski is a good and original piece of mathematics. A major part of it has already been published on important mathematical journals, and so it passed the cross evaluation of other mathematicians. I have a

favorable opinion for the thesis defense.

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