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Referee report on Nikodem Dymski's PhD dissertation

The thesis, as expressed in its title, is devoted to conservation laws in modeling collective phenomena: its Author studies and contributes to our understanding of, mathematical models of traffic flow.

1 Description of main results of the thesis

1.1 Chapter 1

Chapter 1 is a gentle introduction to traffic modeling. Some heuristic arguments presented there may be found unconvincing, but the Author successfully leads the reader through the motivations, history and rudiments of modeling traffic flow. The chapter is concluded with an overview of the results obtained by the Author, discussed in more details in the following chapters.

As a side remark, I mention that the sentence 'Traffic flow is anisotropic, namely drivers move in one direction and they are influenced only by cars in front of them.' featuring on p. 3 is too strong: An ambulance approaching from the rear or an aggressive driver honking at us from behind influences the way we drive stronger than an ordinary car in front of us. A more modest sentence saying that the drivers are influenced *mostly* by cars in front of them would be more appropriate.

1.2 Chapter 2

Chapter 2 is commenced with a detailed description of the Lighthill–Witham–Richards (LWR) and the Aw-Rascle–Zhang (ARZ) models. This is followed by a section on phase transition models including the Author's own contribution to this field; this section may be seen as one of the main parts of the thesis.

The main idea is that the LWR model seems to work well in free regimes, i.e., roughly speaking, on the roads with low density where cars may drive with nearly maximal speed. On such roads one of the system variables may be seen as a function of the other. On the congested flow roads, on the other hand, this is not the case and the master equation has different characteristics. One is thus naturally lead to a system of PDEs with one of the components having radically different properties and nature than the other. The Author considers both the case where the phases intersect non-trivially and the case where their intersection is empty, presents an explicit formula for a Riemann solver in either of these cases, and briefly discusses its properties.

The formulae are quite involved, containing complex notations and numerous cases and sub-cases to the point of being incomprehensible to a normal non-desperate human[1]. I envy the Author and his advisors that they were able to come up with such formulae and efficiently handle them. All the relevant proofs, being yet more technical, are deferred to Appendix A called, for a reason unknown to me, Section A.

Let me end this subsection with a comment that, being not a specialist, I find, at least initially, the following sentence counterintuitive: 'We recall that roads with entrances or exists[2] could be considered by adding a source term, see for instance [8] and the references therein' (see p. 15). To explain: if we imagine the road as a unit interval $[0, 1]$, and the featuring cars as moving in the positive direction, then new cars should show up at the entrance $x = 0$ and existing cars should vanish upon arriving at $x = 1$, and such phenomena are usually described by appropriate boundary conditions. The Author states, however, that a typical way to model this is via a source term. It is my understanding that a source term describes creation (or annihilation) of objects in the interior of the state-space, i.e. of the interval $[0, 1]$. In other words, a source term would describe *a continuum* of other roads via which new cars may enter the road under consideration and/or a continuum of roads through which cars may exit the road under consideration.

Strictly speaking, this is not a remark on N. Dymski's thesis, because the thesis is restricted to roads without entrances and exits. Nevertheless, the way entrances and exists are incorporated into the models existing in the literature, including the models presented in the thesis, pertains to and speaks of the nature of these models, and in particular of a peculiar manner in which they approximate quantities that are discrete by nature by their continuous counterparts. But perhaps discussing philosophy behind the contemporary approach to traffic modeling would lead the Author too far from the main subject of the thesis.

[1] Including mathematicians.

[2] So in the original; this should be 'exits'.

1.3 Chapters 3 and 4

The short Chapter 3 presents the so-called constrained LWR model, taking into account a single obstacle on the road, like traffic lights or a construction site. This chapter is followed by the discussion of the analogous constrained ARZ model in Chapter 4. The main (new) theoretical result of Chapter 4, preceded by relevant historical comments, says that there is a non-fully conservative weak solution to the Cauchy problem related to the constrained ARZ model in the class of functions of bounded variation.

In the main argument, the Author uses the so-called Wave Front Tracking Method, providing a way to approximate the searched-for solution and, as a key to the subsequent analysis, finds a common bound for the total variation of the approximating functions. This allows using the Helly principle to infer convergence and then to show that the limit function is indeed a weak solution to the original problem.

I am not quite sure why Chapter 4 is titled 'Constrained ARZ models'; it seems to me that only one ARZ model is discussed here.

1.4 Chapter 5

As a logical consequence of the material presented in the previous chapters, Chapter 5 is devoted to a model with phase transition accompanied by an obstacle/constraint. Initially, the Author studies the cases where (a) the intersection of free and congested phases is non-trivial and (b) this intersection is trivial. Then, he proves an existence result for solutions of the related Cauchy problem in the first of these cases.

Even though the proof is in nature rather similar to that of Chapter 4, as it hinges again on Helly's Theorem, it is, not surprisingly in view of entanglement of the situation at hand, substantially more involved and much, much longer, even if some more technical points are deferred to the appendix.[3] Incidentally, intricacy of the arguments presented in this chapter illustrates well the fact that, as I am convinced, the Author is well-aquinted with the techniques and tools of modern analysis. A proof of this complexity requires some maturity and being conversant with the techniques used (at the level, or perhaps above the level of PhD), of course under guidance of promotors of the thesis.

1.5 Chapter 6

Chapter 6 adds a new dimension to the analysis by introducing traffic on networks. More specifically, the Author considers traffic on a directed graph with edges incoming to and outgoing from a single node. (By the way, Author's claim that 'the general case is analogous' is either too bold or too vague. Is traffic on

[3] Certainly, some sort of compactness is a typical basis for proving existence theorems for differential equations, regardless of whether they are ordinary differential equations or stochastic partial differential equations, but the similarities between the techniques used in Chapters 3 and Chapter 5 go far beyond that superficial remark.

all graphs really analogous to that on a simple graph with one node?). The main Sections 6.2 and 6.3 of this chapter are devoted to an LWR model with a moving bottleneck (i.e., a model with, say, a slow truck, moving along the road and thus blocking the traffic) and to phase transition models on this simple graph, respectively. Notably, the model of Section 6.2 involves coupling of a PDE with an ODE; such systems are still of much interest and are known to pose new challenges.

2 The language of the thesis

The thesis is written in English. Whereas exercising this skill is recommendable for a young scientist, I was surprised to see that, in the time when nearly every young, aspiring person in Poland is exposed to and speaks English, the candidate's command of this language is rather limited. In particular, in several places I have found that he uses English terms and phrases in the meaning quite different to that they are meant to convey, or in situations where they cannot be used legitimately. Here are some typical examples.

(i) On page 111, the proof of Proposition 6.1 requires considering several cases. Having discussed of one of these cases, the Authors writes 'On the contrary, if ... '. Now, according to the Oxford Dictionary of English this phrase is 'used to intensify a denial of what has just been implied or stated by suggesting that the opposite is the case'. Put simpler, it is used in sentences like 'This number is not positive; on the contrary, it is smaller than -8' and thus corresponds well to the Polish word 'przeciwnie'. Therefore, the Author could have used 'Moreover', 'On the other hand', 'Also', or the like, depending on his taste and the meaning he had in mind, but definitely not 'On the contrary'. See also pp. 35 and 42, where the same mistake is repeated.

(ii) On page iv, the Author writes 'This is caused by two main difficulties. The former is (...). The latter is (...).' Now, by definition, 'former' means 'preceding in order, being the first of the two' and the 'latter' is 'the second mentioned of two as distinguished from former'. These adjectives are used in constructions like 'The set $\mathbb{R}$ is composed of rational and irrational numbers; the former numbers can be expressed in the form $\frac{m}{n}$ where m and n are integers, the latter cannot.' If a previous sentence does not mention two objects in a specified order, one simply cannot say 'the former' and 'the latter' since this is ambiguous and points to nothing in particular. The correct way to start the second and the third sentences in the example at hand is 'The first is' and 'The second is' because these two expressions can be used in a more general situation, where no particular order is a priori given. The Author repeats his mistake on pp. 29, 45, 51 and at two other instances.

(iii) On pages 8 and 106, the Author uses a non-existing word 'informations'.

In English, in contrast to Polish, 'information' is a non-countable noun, so one cannot have many 'informations'.

(iv) The expression 'At last' used numerous times in the thesis seems to be similar to the more commonly used in mathematical texts 'Finally', but in fact these two are quite different. 'At last' means 'after much delay' and is used when there has been impatience, a discomfort or a feeling which was the result of long delays, like in the sentence 'You have cleaned your room at last'. 'Finally' is more suitable in mathematical reasonings since it suggests that to come to this point a long action or a long, involved argument was needed.

(v) On page 15, the Author starts a sentence with 'Whatsoever'. Judging by the context, I guess he intends to say something like 'Nevertheless' or 'This notwithstanding'. He apparently does not realize that 'whatsoever' is used for emphasis in sentences like 'I have no clue whatsoever what a stochastic differential equation is', and is close to the Polish 'zupełnie nie, absolutnie nie, w ogóle nie'. Similarly, on p. 12 'For more clarity' is perhaps meant to convey the meaning of 'More specifically'.

There are also numerous places where the definite and/or indefinite articles are missing or are used incorrectly[4], sentences like

> *This two-phase approach was first introduced by Colombo in [28] and after exploited by other authors*[5]

misspelled words (e.g. 'wheter' instead of 'whether' and 'riemann' instead of 'Riemann'), various errors, like that on p. 10:

> *Nevertheless, none of the discussed construction is free of drawbacks. One of the main issue raised about the LWR model is infinite acceleration.*[6]

and other issues, I would rather not discuss here[7], including (mis)usage of commas, parentheses and quotation marks.

I refrain myself from presenting the entire list of such mistakes, errors, misprints, typos and unintended semi-comical effects[8][9]. They definitely are hin-

[4]For example, 'a probability distributions' on p. 3, and 'the condition (A.x)' several times on p. 4.

[5]Instead of *This two-phase approach was first introduced by Colombo in [28] and later exploited by other authors.*

[6]This should perhaps read *Nevertheless, none of the discussed constructions is free of drawbacks. One of the main issues raised about the LWR model is infinite acceleration.*

[7]For instance, 'Let place an observer', 'does formula holds true', 'which turns out the model non-realistic', 'the goal is obtained', 'the main theorem distinguish two cases','equation does not match to the reality', 'piecewise function', 'domains (...) refers', 'the states (...) represents', 'one of the following condition', 'let assign'.

[8]Like 'numerous number of models' on page 3.

[9]Neither do I want to discuss issues related to the part written in Polish, like 'Dla Agaty' instead of 'Agacie' and hyphenation of Polish words. But I must say that the English 'proposition' and the Polish 'propozycja', even though they look similar, do not have the same meaning.

drances to reading the thesis, but the entire text, with some goodwill, is readable, intelligible and comprehensible. In fact, whereas the Author's command of English is limited, his knowledge of the field of mathematics in which he specializes is impressive, perhaps exceeding a typical level of a PhD student.

3 Conclusion

It is my conviction that Nikodem Dymski's dissertation is devoted to an important subject of current interest. Notwithstanding the shortcomings of the thesis, his work contributes to our understanding of traffic flow, as seen from the mathematical point of view. Moreover, preparation of the thesis required mastering substantial parts of modern analysis, including the theory of partial differential equations, its ideas, notions and tools. In addition, I find that Nikodem Dymski, definitely led in this respect by the promotors of his thesis, knows the relevant literature quite well and adequately understands the general picture and more subtle aspects of the field he works in.

Therefore, taking into account the work needed and the goals achieved, I am persuaded that the Author has a right to aspire for the PhD title: I am in favor of allowing him to defend his thesis.

Adam Bobrowski