PHD THESIS SUMMARY

ADAM GĄGOL

The following dissertation is devoted to the applications of a probabilistic method in proving combinatorial theorems on pattern avoidance. The probabilistic method in combinatorics is a special way of using theorems of the probability theory to prove the existence of mathematical objects that fulfill certain properties. In its basic version, it assumes the construction of an appropriate probabilistic space and proof that a randomly selected object from this space probability has the desired properties with non-zero.

An important result of the probabilistic method is the Lovász Local Lemma, which allows to prove the existence of structures that avoid the collection of local properties. Among the many versions of the lemma, there is an algorithmic version, in which an object that avoids all undesired local properties is constructed using a simple Las Vegas algorithm with a polynomial expected duration of operation. The algorithm operates on the assumption that all A events are determined by a finite number of independent random variables P. In the general form, the algorithm is as follows:

In the first step, the algorithm assigns random values to all random variables $p \in P$. Then, in the main loop, it randomly changes the values of random variables that make up the occurring events of A until a situation in which none of the events occurs.

Apart from the obvious advantage of this approach, i.e. the direct construction of objects satisfying the given set of constraints, it also allows to improve some of the estimates resulting from the non-algorithmic versions of the lemma. This is possible thanks to the use of relations between random variables p and avoided events A, what escapes the classic version of the lemma.

The two results presented in the thesis concerns the avoidance of patterns in words (chapter 2) and graphs (chapter 3) make a significant use of this relation and the algorithmic version of the lemma.

The first theorem presented in the dissertation is the result of the author's independent work and concerns the avoidance of patterns (i.e., such strings in which individual substrings can be assigned to successively occurring variables in the pattern) in partial words.

Theorem. *If p is a pattern with $m > 2$ variables such that $|p| \geq 2^m$, then p is $*$-avoidable over the ternary alphabet.*

The concept of $*$-avoidability is a natural extension of avoidability to partial words. This theorem approximates the full classification of patterns to those that are avoidable in partial words over the binary and ternary alphabet and those that are not avoidable over any finite alphabet, leaving a problem open only for three patterns with two variables.

The second result presented is the proof of the theorem:

Theorem. *There is a function f such that if T is a tree with the pathwidth w then there exists an nonrepetitive coloring T from any lists of length $f(w)$.*

Moreover, we present an example of graph class with a path width of 2 for which there is no finite k such that each of the graphs in this class can be colored from lists of length k. This theorem was proved in cooperation with Jakub Kozik, Piotr Micek and Gwenaël Joret.

20.03.2018

Adam Gągol