Seminarium Zakładu Zastosowań Matematyki

21.11.2018, godz. 10:15, s. 240

Turán and Ramsey numbers for 3-uniform path of length 3

Eliza Jackowska-Boryc

Extremal sets theory is an area of discrete mathematics which is concerned with the existence of well-ordered subsets in large sets and plays an important role in many branches of mathematics, for instance in combinatorics, number and game theories, geometry, and logic.

Major questions in the extremal hypergraph theory are related to determining Turán numbers and Ramsey numbers. Determining Ramsey numbers is one of the most difficult problems related to Ramsey's theorem (Graham, Rothschild, Spencer, 1980). In this presentation I will focus on hypergraph Ramsey numbers. Classical Ramsey numbers are hard, and very little is known. For this reason, research in hypergraph Ramsey theory was directed to other structures with less density, for instance, hyperpaths or hypercycles.

There are several definitions of k-uniform paths and cycles. In this presentation we focus on the symmetric case when the intersections of consecutive edges have a fixed size equal to 1. *A loose path* P_l^k is a k-graph with l edges $e_1, ..., e_l$ such that $|e_i \cap e_j| = 0$ if $|i-j| > 1$ and $|e_i \cap e_j| = 1$ if $|i-j| = 1$. Here I focus mainly on 3-uniform loose path of length 3, P_3^3. As a helpful tool, in our proofs we often use a 3-uniform cycle of length 3, C_3^3, which is called the triangle.

I start with a presentation of basic terminology and definitions, and a review of selected earlier results on hypergraph Ramsey and Turán numbers. Then, I introduce my main contribution to Ramsey Theory, which asserts that $R(P_3^3; r) = r + 6$, for the number of colors $r \leqslant 7$, with a proof. The proof of the lower bound is based on a construction given by Gyárfás and Raeisi (2012). To show the upper bound we need the ordinary Turán number $\mathrm{ex}_3(n; P_3^3)$ for all n and other variants of Turán number like conditional Turán number and the Turán numbers of higher orders. Moreover I present the methods and tools which were used in the proofs of considered theorems.

At the end of my presentation I say a few words about the newest results obtained in this field.