

## RAMSEY NUMBERS FOR GRAPHS.

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For any graphs  $H_1, H_2, \dots, H_m$   $m$ -colored Ramsey number  $R(H_1, H_2, \dots, H_m)$  is the smallest natural number  $n$  such that in any  $m$ -edge coloring of complete graph  $G = K_n$  there exists  $i$  ( $1 \leq i \leq m$ ) such that the graph  $G$  contains an isomorphic subgraph of  $H_i$ , which all edges are colored in  $i$ .

The dissertation has been divided into 4 chapters. In the chapter 1 we discuss the problem of determining the exact values of Ramsey numbers and introduce the necessary markings and definitions.

The chapter 2 presents current achievements in this field. We present selected known results for two-color and multi-color Ramsey numbers. We concentrate heavily on numbers whose cycle is one parameter. More interesting results can be found in the regularly updated review paper *S. Radziszowski, Small Ramsey numbers, The Electronic Journal of Combinatorics, Dynamic Survey 1 (2017)* [3].

In the chapter 3, we discuss the results for two-color Ramsey numbers for generalized fans and wheels. We present Ramsey numbers for certain generalized fans  $K_1 + kP_3$  with cycle. This graph is a special case of the graph  $K_1 + L_n$ , where  $L_n$  is a linear forest of the order of  $n$  with at least one edge. We generalize the Shi's [4] theorem for fans and cycles. The contents of the chapter 3 were included in the published paper [1] *H. Bielak, K. Dbrowska, The Ramsey numbers for some subgraphs of generalized wheels versus cycles and paths, Annales UMCS, Sectio A, Mathematica (2015), 69, no. 2, 1–7*. The results of this paper were quoted in paper *Radziszowski, Small Ramsey numbers, The Electronic Journal of Combinatorics, Dynamic Survey 1 (2017), 29–30* [3]. The presented results were presented at the conferences *23rd Workshop 3in1 2014* in Krakow and *Seventh Cracow Conference on Graph Theory* in Rytró.

In the chapter 4, we concentrate mainly on multicolored Ramsey numbers for certain linear forests and cycles. We show that the Turán numbers are useful for determining the upper bound for Ramsey numbers. Selected results presented in this chapter have been included in the paper [2] *H. Bielak, K. Dbrowska, Multicolor Ramsey numbers for cycles versus some sequences of disjoint short paths*, submitted. The presented results have been presented at a few conferences, for example *Seventh Czech-Slovak International Symposium on Graph Theory, Combinatorics, Algorithms and Applications in Kosice, The 16th International Conference Random Structures and Algorithms* in Poznan, *5th Polish Combinatorial Conference* in Bdlewo and *The Fourth Gdask Workshop on Graph Theory* in Gdansk. Below is a partial list of publications.

## References

- [1] H. Bielak, K. Dbrowska, The Ramsey numbers for some subgraphs of generalized wheels versus cycles and paths, *Annales UMCS, Sectio A, Mathematica* (2015), 69, no. 2, 1–7.
- [2] H. Bielak, K. Dbrowska, Multicolor Ramsey numbers for cycles versus some sequences of disjoint short paths, rkopis (wysany do recenzji).
- [3] S. P. Radziszowski, Small Ramsey numbers, *The Electronic Journal of Combinatoric, Dynamic Survey 1* (2017)
- [4] L. Shi, Ramsey numbers of long cycles versus books or wheels, *European J. Combin.* 31 (2010) 828-838.