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SUMMARY PHD THESIS

Compactness conditions in the space of functions with tempered increments and their application in the theory of integral equations

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The PhD dissertation is devoted to the so-called compactness conditions for the space of functions with increments tempered by a given modulus of continuity $C_\omega(M)$. This space consists of real functions, defined on a limited metric space and having increments tempered by a given modulus of continuity . It turns out that the space $C_\omega(M)$ is a Banach space with the norm defined in a natural way, referring directly to the definition of the [4].

Special cases of such a space are the space of real functions which are defined on a given closed and bounded interval and meet at that interval Lipschitz condition and the space of function space that meet Hölder condition at that interval. The above-mentioned items of the space of functions with increments tempered by a modulus of continuity undoubtedly suggest the usefulness of the applications.

Although these spaces are widely known, they are rarely used in considerations of nonlinear analysis. In particular, it is difficult to meet the claims of the existence of solutions of certain equations (differential, integral) in such spaces. This is caused mainly by the fact that in these spaces there are no criteria for the relative compactness or even non-trivial conditions sufficient for the relative compactness.

The submitted paper is supposed to fill this gap and create methods and tools that will allow free use of the above described space of functions with increments tempered by a given modulus of continuity, in particular, the space of Hölder functions.

The main aim of the paper is to provide comfortable compact conditions which can be used to decide on the relative compactness of a fairly broad class of subsets in the above-mentioned space.. In addition, on the basis of the formulated conditions sufficient for the relative compactness of the set, we will present the design of a collective function, which allows quite

skillful use of these conditions sufficient for the relative compactness of the set. This function is called the measure of non-compactness.

It is worth mentioning that in the last forty years the problems associated with measures of non-compactness have been intensively studied by mathematicians from many centers around the world [2, 3]. The scientists have discovered many possible applications of the theory of measure of non-compactness to the various issues considered in the nonlinear analysis, the geometry normed spaces, the theory of operators and the theory of differential and integral equations [1, 2, 3].

The second main objective of the dissertation is a comprehensive examination of the constructed measure of non-compactness and presentation of its properties with the evidence.
In addition, we show the usefulness of the constructed measure of non-compactness in the proofs of existential theorems for non-linear integral equations in the considered areas of Hölder functions. Integral equations that will be discussed in this paper are the square integral equation of Hammerstein type and the square integral equation of Volterra-Hammerstein type. They are formed by multiplying integral operator by the so-called operator of superposition. However, the operator of superposition considered in this paper is a linear operator, and even, strictly speaking, the affine operator. Such narrow considerations concerning the operator of superposition that acts as a multiplier of integral operators are mainly due to the final outcome presented by J. Matkowski [6, 7].

Most of the results presented in the submitted dissertation were published in the papers [4, 5].

References

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