

Abstract of the PhD thesis
New properties of determinants of rectangular matrices
and their geometric interpretation.

The PhD thesis „*New properties of determinants of rectangular matrices and their geometric interpretation*” contains the results of three scientific papers of which one is published, one is accepted for publication, and one is being prepared:

1. A. Makarewicz, P. Pikuta, D. Szalkowski, Properties of the determinant of a rectangular matrix, *Ann. Univ. Mariae Curie-Skłodowska Sect. A*, vol. 68, no. 1, 31-41, 2014,
2. A. Makarewicz, W. Mozgawa, P. Pikuta, Volumes of polyhedra in terms of determinants of rectangular matrices, accepted to printing in *Bulletin de la Société des Sciences et des Lettres de Łódź. Série: Recherches sur les Déformations*, vol. 66, no. 2, 2016,
3. A. Makarewicz, P. Pikuta, How appending duplicate columns of a matrix affects its determinant - in preparation.

The results of the above papers are presented in chapters 2 and 3 and they are divided in natural way on the algebraic part and geometric part. Furthermore the PhD thesis includes introduction and overview of the results that have appeared so far in terms of determinants of a rectangular matrix, in which the list of selected definitions of the determinant of a rectangular matrix and the list of theorems required in the following chapters are presented.

In the second chapter, the algebraic properties of the determinants of rectangular matrices are presented. There are given several theorems, some of which describe properties similar to the ones corresponding with the determinant of a square matrix. These are:

1. formulas for the determinant of a rectangular matrix, which express the determinant by using the sum of the determinants of square matrices, but in a different way than in the definition,
2. formula describing the transformation of the determinant in the case where a column of the matrix is the sum of the other columns.
3. the formula describing the transformation of the determinant under the exchange of columns of the matrix,
4. formulas describing how the determinant of a rectangular matrix changes, when instead of one column we put a certain number of copies of this column, when instead of a pair of columns we put a certain number of copies of this pair of columns and in general - when, instead of all the columns of the matrix we put a certain number of copies of these columns.

In the third chapter, issues related to the geometric interpretation of the determinants of rectangular matrices are considered. This topic has not been studied enough so far in mathematical literature. By studying the geometric interpretation of the determinant of a rectangular matrix which has m rows and n columns, the columns of the matrix as different points of the space \mathbb{R}^m are considered. In the following sections of this chapter a few theorems are proved which yield formulas that can be used to calculate:

- a. the volume of an octahedron,
- b. the volume of a tetragonal pyramid
- c. the volume of a triangular prisms and a truncated triangular prisms.
- d. The area of a parallelogram and the area of a triangle.

The achieved results are an original contribution to the subject of the determinants of rectangular matrices.

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